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## « Contents »




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THE MATHEMATICS TEACHER

Volume XXVIII Number 6

Edited by William David Reeve

# Third Report of the Committee on Geometry

*By* Ralph Beatley
*Harvard Graduate School of Education*
*Cambridge, Massachusetts*

The first two reports of the Committee on Geometry of the National Council of Teachers of Mathematics published in *The Mathematics Teacher* for November 1932 and October 1933 gave a list of the membership of the committee and a plan of action having as its goal the stimulation of experiments in the teaching of geometry by classroom teachers with a view to improving instruction in demonstrative geometry and making it mean more to the pupils.

The first step in this program was to collect the ideas of many teachers of mathematics here and in Europe concerning geometry, especially those ideas which have not been broadly disseminated and are not exemplified in present practice in this country. This quest for the novel and the foreign indicates no bias on the part of the committee in favor of strange and untried doctrines, but merely a desire to encourage the consideration of all suggestions that seem to have merit or backing. A sub-committee of three[1] examined a long list of articles in periodicals, reports of committees, books on method, and school textbooks, domestic and foreign, and prepared

[1] Miss Martha Hildebrandt, Albert J. Schwartz, and Ralph Beatley.

synopses recording the more important suggestions to be gleaned from these sources. These synopses appear as an appendix to this report, Appendix A.

The second step in the committee's program was to study the ideas contained in these synopses, to record its own opinion concerning them, to get the opinions of other teachers also, and then to select those ideas which seemed to hold most promise of success if tried in our own schools. These last are presented later in this report in the form of a few variant philosophies of geometry which can serve as bases for experiments to be conducted by classroom teachers who are interested in improving the teaching of geometry. It is thought that many would be glad to try experiments on a small scale if they could envisage their experiment as part of a broad program, and that administrative officers would welcome suggestions from this committee as to procedures worth trying and as to programs holding most promise of success. As one means of justifying further consideration by school officers and teachers of mathematics of the place and function of demonstrative geometry Dr. Vera Sanford consented to prepare an article for publication presenting anew the possibilities of this subject in the light of recent criticism.[2]

This committee can do little more than to suggest and to encourage certain avenues of experiment. If funds were available—say, several hundred dollars—it could coordinate experiments wherever such coordination might be acceptable, it could offer through correspondence the counsel of an expert in educational experimentation, and it could collate and publish the results of these experiments. Even without funds it would seem worth while at least that this committee on geometry be kept for a few years as a standing committee to serve as an informal clearinghouse for ideas, queries, and suggestions.

Let us return to the work of the committee already accomplished and learn its opinions and recommendations. The major ideas turned up by the sub-committee in its preparation of the synopses in Appendix A are listed below. There are, of course, incompatible items in this list. A questionnaire based on these ideas, Appendix B of this report, was sent to each member of the committee, and several members distributed copies to regional associations of

[2] See *The Mathematics Teacher*, May 1935.

teachers of mathematics, so that the replies represent not only the opinions of the members of the committee, but the opinions of 101 teachers in Maine, Massachusetts, Ohio, Illinois, Minnesota, Kansas, Oklahoma, and Colorado as well. On the questionnaire in Appendix B are summarized the replies of the members of the committee. A concomitant summary of the replies of these 101 other teachers makes it easy to compare the opinions of the two groups; there are few significant differences between them.

The major ideas derived from the synopses, and on which the questionnaire was based, were the following—some of them incompatible, as pointed out above:

1. *The important facts of geometry can be learned below the tenth grade, in informal geometry.*
2. *The informal geometry of the junior high school ought to include measurement, constructions, inductions based on measurement, inductions based on observation, and simple deductions based on geometric inductions and intuitions.*
3. *We ought to recognize three stages in our instruction in geometry, informal, transitional, and systematic.*
4. *The work of grades 5 and 6 ought to include some informal geometry.*
5. *The course in informal geometry and demonstrative geometry ought to be spread over four or five years in the junior and senior high schools.*
6. *Demonstrative geometry in grade 9 should serve as transition from informal geometry to systematic geometry.*
7. *Demonstrative geometry in grade 9 should be the first stage of the regular systematic treatment of geometry.*
8. *Instruction in demonstrative geometry ought to be limited to students of 110 I.Q. or better, i.e. to the upper half of the total population of the tenth grade.*
9. *Demonstrative geometry can be adapted to suit all but the lowest levels of intelligence.*
10. *Bright students in industrial, commercial, and home economics curricula ought not to be dissuaded from studying demonstrative geometry.*
11. *The main outcomes of demonstrative geometry pertain to logical thinking.*
12. *We wish pupils to develop the conscious use of a technique of thinking.*

13. *Demonstrative geometry ought to call attention to logical chains of theorems; to gaps in Euclid's logic; to the nature of a mathematical system, the need of undefined terms, the arbitrariness of assumptions, and the possibility of other arrangements of propositions than that given in any one text.*
14. *The educational possibilities of demonstrative geometry depend upon the "transfer" of the logical training of geometry to situations outside geometry. We ought to teach so as to insure this transfer.*
15. *Do we wish to preserve our Given-To Prove-Proof formalism, or to replace it by an equally rigorous but more informal mode of expression?*
16. *Demonstrative geometry is concerned with practical applications as well as with logic.*
17. *There is little profit in viewing geometry as a logical system.*
18. *Proof by superposition is mathematically valid but psychologically undesirable at the beginning of demonstrative geometry.*
19. *We ought to postulate all propositions requiring proofs by superposition.*
20. *We ought to postulate all propositions that seem obvious to the pupil.*
21. *Propositions having traditionally difficult proofs can be assumed without proof in a first view of the subject; later the pupil can return to them and prove them readily.*
22. *We ought to postpone rigorous treatment of congruence and make the theorems concerning parallel lines serve as the first approach to rigorous demonstration.*
23. *We ought to begin geometry with proportion, linking this with the ratio of irrational numbers.*
24. *We ought to begin with originals instead of book theorems.*
25. *All book theorems should be treated as originals.*
26. *The distinction between book theorems and originals on college entrance examinations ought to be abandoned.*
27. *The logical structure of individual theorems should be revealed by analysis before the synthetic proof is displayed.*
28. *The indirect method is the recognized geometrical method for converse theorems and cannot be entirely avoided.*
29. *We ought to make free use of translation, rotation, and symmetry in proving theorems.*
30. *We ought to assume explicitly that symmetric figures are equal (congruent).*

31. *It would be desirable to postulate one or two propositions on congruent triangles; the propositions concerning vertical angles, the unique perpendicular, and parallel lines and corresponding angles; the propositions concerning central angles and their corresponding arcs and chords; the proposition concerning the line which divides two sides of a triangle proportionally; the proposition that the area of a rectangle equals base times altitude; and the like.*
32. *The theorems concerning inequalities could be omitted.*
33. *The usual order of the ideas congruence, parallelism, similarity could profitably be altered to the order congruence, similarity, parallelism.*
34. *Incommensurables ought to be omitted.*
35. *A free use of irrational numbers would make possible a simple treatment of incommensurables.*
36. *The notion of limit ought not to be lost from geometry.*
37. *We should treat proportion algebraically.*
38. *Legendre assumed the correspondence between line segment and number and based part of his geometry on arithmetic and algebra.*
39. *Algebraic methods ought to be used freely.*
40. *It is desirable to link trigonometry with the study of similar triangles.*
41. *We ought to restrict proofs of loci to seven basic theorems.*
42. *Pertinent reference ought to be made to solid geometry during the course in plane geometry.*
43. *The content of elementary geometry should be enriched by adding a few simple ideas from "modern" geometry; for example, radical axis, center of similitude, central projection, inversion, duality.*
44. *The idea of functionality ought to be prominent in demonstrative geometry, both in the factual aspects and in the logical aspects of the subject.*
45. *The usual year and a half devoted to plane and solid geometry ought to be reordered on the basis of difficulty rather than dimension.*
46. *In solid geometry we ought to lay less stress on logical demonstration, except for certain significant groups of theorems, and make greater application of trigonometry to three dimensional figures.*
47. *More originals in geometry ought to be based on situations from surveying, from physics.*

The replies to the questionnaire based on these major ideas ap-

pear in Appendix B of this report. Their general tenor is indicated by the following summary.

It is generally agreed that the important facts of geometry can be mastered below the tenth grade through inductions based on observation, measurement, constructions with drawing instruments, cutting and pasting, and also through simple deductions from the foregoing inductions as well as from geometric notions intuitively held. A strong majority favor the construction by junior high school pupils of the common geometric solids, with the exception of the regular dodecahedron and icosahedron, and favor also an extension of the usual work in mensuration to include consideration of the ratios of corresponding lengths, areas, and volumes of similar solids. The suggestion that this informal solid geometry include elementary ideas concerning parallel planes cut by transversals, plane sections of solids, perpendicular lines and planes is opposed by a slight majority of the committee and favored by a slight majority of the 101 other teachers—almost a fifty fifty division on these items, and on the item concerning longitude and time also.

The replies to the question concerning the course of study in geometry are to some extent contradictory and not easily summarized. Teachers incline on the whole to concentrate the informal geometry in one grade of the junior high school, showing a slight preference for the eighth grade though with many exceptions in favor of the seventh. There is more definite preference for the concentration of demonstrative geometry in a single grade of the senior high school. Teachers agree that the main outcomes of demonstrative geometry pertain to logical thinking and wish to maintain the distinction between this subject and informal geometry, which emphasizes the factual aspects of geometry. They do, however, favor a gradual transition from fact to proof, but prefer on the whole to get this by adding simple deductions to the informal stage than by constructing a separate middle stage to serve as transition.

The proposal that the pupil begin demonstrative geometry by attempting to prove simple originals instead of book theorems, and that the logical structure of individual propositions be emphasized by requiring analyses wherever possible before the synthetic proofs, receives almost universal support. There is equally enthusiastic response to the proposal that instruction in demonstrative geom-

etry call attention to logical chains of theorems, to the gaps in Euclid's logic, and bring the pupil to appreciate the nature of a mathematical system, the need of undefined terms, the arbitrariness of assumptions, and the possibility of other arrangements of propositions than that given in his own text. In accord with this sentiment is the almost unanimous vote in favor of widening the list of assumptions so as to include theorems the truth of which seems incontestable to the pupil. A large majority favor also the suggestion that proof by superposition of the first two theorems on congruent triangles, and of the theorems concerning central angles and their corresponding arcs, be avoided by postulating these propositions.

The proposal in the British Report on The Teaching of Geometry in Schools[3] that the parallel postulate might better be supplanted by a postulate of similarity receives approval from only one-fourth of the committee, and from one-fourth of the 101 other teachers also. This proposal, which amounts in effect to assuming the first theorem on similar triangles and placing it early in the course, is designed to give equal prominence to the ideas of similarity and congruence, and to assign a lesser role to parallelism. Your committee, on the contrary, prefers to postulate one or both propositions concerning corresponding angles and parallel lines; some few indeed would make parallels serve as the very beginning of demonstrative geometry, before considering congruence.

A large majority favor the free use of algebra in demonstrative geometry and approve some attempt to link incommensurables with irrational numbers. There is general unwillingness to ignore incommensurables entirely in the instruction; probably few, however, would urge the suitability of this topic for examination purposes. Both in the committee and among the 101 other teachers a considerable majority, but not a large majority, are opposed to inserting a treatment of numerical trigonometry in connection with similar triangles, preferring to cover this subject earlier in informal geometry or in connection with algebra.

Nearly everyone favors occasional pertinent reference to solid geometry, mainly of a mensurational sort, in the course in plane geometry. Indeed, one-third the committee desires to merge plane and solid even more intimately, and the 101 other teachers are about equally divided on this point. A slight majority in the com-

[3] G. Bell and Sons, London.

mittee favor enriching the content of the exercises by introducing a few simple ideas from "modern" geometry such as radical axis, center of similitude, central projection, inversion; of the 101 other teachers a majority oppose this suggestion except for superior pupils.

There is almost unanimous agreement that demonstrative geometry can be so taught that it will develop the power to reason logically more readily than other school subjects, and that the degree of transfer of this logical training to situations outside geometry is a fair measure of the efficacy of the instruction. However great the partisan bias in this expression of opinion, the question "Do teachers of geometry ordinarily teach in such a way as to secure the transfer of those methods, attitudes, and appreciations which are commonly said to be most easily transferable?" elicits an almost unanimous but sorrowful "No."A majority declare that for students in technical, industrial, commercial, and home economics curricula the logical aspects of geometry are at least as important as the factual outcomes. This is not to be interpreted, of course, as meaning that demonstrative geometry is equally desirable for students in these different curricula, but merely that those who elect to study geometry ought to derive about the same benefits from their study, regardless of curriculum. When pressed to define what is meant by the application of the function concept to geometry, most of the committee come to the conclusion that this refers to the factual aspect of geometry and not to the logical side of the subject. For the most part they have in mind the dependence of certain numbers used to measure angles, lengths, areas, and volumes upon certain other numbers of a similar sort. Only a very few would stretch the idea of functionality to include the idea "varying the hypothesis varies the conclusion."

The list of major ideas and the responses to the questionnaire suggest the following six experimental programs, Iabc, Iac, Ibc, II, III, IV, based on four different philosophies of geometry. Each of these four philosophies and the six programs based on them is compatible with the general philosophy of secondary education in this country. Certain ideas covered by the questionnaire which are consonant with all these programs and therefore open equally to separate experiment or to merger with any of the experimental programs I–IV, are cited in the independent experimental programs Va, Vb, Vc, Vd, Ve.

*Four Variant Philosophies of Geometry Exemplified in Six Experimental Programs for Secondary Schools: Iabc, Iac, Ibc, II, III, IV*

All these programs envisage a certain amount of informal geometry in the junior high school. Whether or not this informal geometry shall be extended into the early part of grade 10 depends upon the amount covered in the junior high school and is a matter for individual schools to decide. Present geometry texts include introductory material of this sort.

These programs all differ in some respect from usual practice in the schools. This is not to be interpreted as a blanket condemnation of all present practice; these programs merely indicate various avenues of inquiry into the possibility of improvement.

*Program Iabc*

(a) An introductory section clearly separating the factual material of geometry from the logical aspects.

(b) A brief introduction to the logical aspects of geometry, including discussion of the need of assumptions and undefined terms, form of proof, analysis, indirect method, converses, and begging the question.

(c) A logical treatment of demonstrative geometry similar to that commonly given in the United States but with a wider list of assumptions, including at least one of the propositions on congruent triangles; the propositions concerning vertical angles, the unique perpendicular, and parallel lines and corresponding angles; the propositions concerning central angles and their corresponding arcs and chords; the proposition concerning the line which divides two sides of a triangle proportionally; the proposition that the area of a rectangle equals base times altitude; and the like. Including definite mention also of the few inevitable gaps in the logical system.

The time necessary for Ib can be gained by condensing the logical treatment somewhat by assuming certain propositions as noted in Ic.

*Program Iac*

Same as Iabc except that the purpose and emphasis of Ib, itself brief, are reduced to the even briefer treatment accorded these topics in most schools today.

*Program Ibc*

Same as Iabc with the omission of Ia, indicating not so much a lesser interest in an informal approach to geometry as an insistence that it be accomplished below the tenth grade.

*Program II*

(a) A brief introduction to the logical aspects of geometry as outlined in Ib.

(b) A brief, loosely knit geometry of the Bourlet or Malsch type corresponding to Stage B of the British Report on the Teaching of Geometry in Schools, designed to show the nature of geometric argument and to serve as transition to a logical, systematic, treatment.

(c) A logical treatment of demonstrative geometry similar to that commonly given in the United States, but with a wider list of assumptions as noted in Ic, and with comment on the few inevitable gaps in the logical system.

Program II puts the emphasis on logic and uses geometry as a vehicle for the logic. In addition to the informal acquisition of geometric facts in grades 7, 8, 9 by observation and experiment, there are two further introductory steps to the systematic course in demonstrative geometry. First, a brief exposition of the nature of all logical systems, as in Ib and IIa. Second, a brief exemplification of a logical treatment of geometry to show what it would look like and to afford an easy transition to the systematic logical development of the subject. In this transitional stage IIb any proposition could be assumed which the pupils were ready to grant without proof as a result of their informal geometry in the junior high school grades. Free use of intuition would be permitted and proofs would be given only at points seeming to the students to require justification. Following these two introductory stages the bulk of the course, IIc, would be a logical treatment of demonstrative geometry only slightly different from present practice in the United States.

Time for the brief exposition of the nature of a logical system (IIa) and for the brief intuitional treatment of demonstrative geometry (IIb) would be gained by relegating all the informal geometry to the lower grades and by condensing the logical treatment somewhat by assuming certain propositions as noted in Ic. This systematic logical treatment IIc, although just like Ic, would

probably require less time because of the understanding and experience gained from the transitional section IIb.

It should be noted that on the questionnaire this committee preferred plan 10e, favoring some deductions in the informal stage and appropriate reference to applications in the logical stage.

*Program III*

(a) A brief introduction to the logical aspects of geometry as outlined in Ib and IIa.

(b) An extension of the transitional stage (IIb) so that it gradually merges with the logical stage (IIc) without attaining the logical form and complete systematization now so definitely a part of our method of presentation in the United States, and also in Italy and Britain. In short, to follow the pattern of Chenevier, Bourlet, Malsch, Lietzmann, Wolff, and other recent standard texts in France and Germany.

This program teaches geometry and regards the logic as incidental; a deficiency which could be made good by appending a brief section in which is devised a logical system fit to comprise the geometry which has been studied. This plan is included because of its vogue in France and Germany to-day.

*Program IV*

First the introductory and transitional stages described above. Then a variant of the systematic logical presentation (Ic, IIc, IIIb) which makes explicit use of the real number system to give a rigorous but more numerical treatment of geometry than is our present practice. An early and powerful assumption in this geometry would be the proposition that two triangles are similar if an angle of one equals an angle of the other and the sides including these angles are proportional. If this proportion is expressed by means of a factor of proportionality $k$, it is clear that congruence appears as a special case in which $k$ equals 1. This introduction of similar triangles and proportion at the outset makes possible an early proof of the most important propositions, including the Pythagorean Theorem, and effects a considerable condensation of the entire geometry. It rests squarely on the pupils' elementary intuitions concerning irrational numbers and makes possible a treatment of geometry broad enough to include incommensurables and commensurables under the same head without necessity of distinguishing one from the other.

*Five Independent Experimental Programs, Va, Vb, Vc, Vd, Ve*

(a) The provision of "transfer" material to facilitate application of the logical discipline of demonstrative geometry and the appreciation of its logical structure to non-geometric situations in real life. Experiments in this direction would be largely independent of the program in demonstrative geometry.

Other independent experiments could examine the desirability of employing (b) algebraic and (c) trigonometric methods in demonstrative geometry, and (d) of introducing certain ideas from "modern geometry." The degree to which pertinent allusion to solid geometry can safely be carried in a first treatment of plane geometry is another fit subject (e) for inquiry.

The members of the committee were asked to rank these several experimental programs in order of promise, putting the most promising first, and to offer any criticism they might feel disposed to make. They were invited to submit substitute programs for any of the foregoing, and additional programs. Their ranking of the several programs is exhibited in the following table, which should

| Program \ Rank | 1 | 2 | 3 | 4 | 5 | 6 | O |
|---|---|---|---|---|---|---|---|
| Iabc | 5 | 4 | 5 | 2 | 4 | 0 | 1 |
| Iac | 1 | 3 | 4 | 6 | 3 | 2 | 2 |
| Ibc | 3 | 3 | 3 | 6 | 2 | 1 | 3 |
| II | 6 | 6 | 1 | 2 | 4 | 0 | 2 |
| III | 2 | 2 | 1 | 3 | 5 | 7 | 1 |
| IV | 4 | 1 | 5 | 0 | 1 | 8 | 2 |
| Va | 13 | 1 | 0 | 2 | 1 | | 4 |
| Vb | 2 | 7 | 5 | 3 | 0 | | 4 |
| Vc | 1 | 1 | 4 | 4 | 6 | | 5 |
| Vd | 0 | 1 | 2 | 4 | 7 | | 7 |
| Ve | 2 | 7 | 6 | 2 | 0 | | 4 |

be read as follows: Program Iabc was ranked first by 5 members of the committee, second by 4 members, and so on. The symbol *O* means "omitted."

It would appear that the committee regards program II as the most promising for immediate experiment, followed in order by programs Iabc; Ibc, Iac; IV, III. The distribution of opinions concerning the promise of program IV is tri-modal, making it difficult to rank it with the other programs. I believe I have placed it where it belongs. The independent programs were ranked as follows: Va; Ve, Vb; Vc, Vd.

The returns indicate a demand for two introductions to demonstrative geometry. First, an informal presentation of the more important facts of geometry; second, an introduction to the logical method of demonstrative geometry. Teachers evidently prefer that this first introduction to geometry be completed in the junior high school grades so that there will be time to expand the introduction to logical methods into a short transitional section. Though willing to enlarge the list of assumptions, they regard as less promising the experiment proposed in program IV, which would rearrange the assumptions and strengthen some of them; and as least promising of all the modern French and German practice of relaxing the logical formalism in favor of a more "rhetorical" presentation.

This desire to preserve the logical side of demonstrative geometry is revealed also by the marked eagerness to experiment with "transfer" material. There is very considerable interest in pertinent reference to three dimensional items, and in freer use of algebraic methods. The introduction of elementary ideas from "modern geometry" and the use of trigonometric methods are regarded as less promising at this time.

Some of the criticisms received from members of the committee ought to be recorded here, especially those which show divergence from the majority opinions expressed above. Professor Court believes that beginners should regard geometry first of all as a physical science, and that the ideas concerning the logical structure of the subject should be presented gradually, with almost no mention of them at the beginning. Other first-rate mathematicians not members of this committee have said the same thing from time to time. It should be remembered, of course, that the informal geometry of the junior high school presents geometry as a physical science, exactly as Professor Court and others advocate, and that

the gradual unfolding of the logical side of demonstrative geometry is not intended to be so severe a discipline as might be inferred from the descriptions of this material. Nevertheless, it is well to be warned against overemphasis of the logical aspects. On the other hand, the opinion is growing among teachers that many of the students' difficulties spring from their failure to appreciate the logical intent of the subject, and that appreciation of what it is all about tends to remove these difficulties. Furthermore, if we do not make every effort to see that our pupils obtain the logical understanding and appreciation so widely heralded as an important outcome of their study of demonstrative geometry, how can we continue to urge these logical aspects in support of our plea for the retention of this subject in the course of study of the secondary school?

Professor Perry suggests that further experiment in geometry ought to be based upon the techniques and results of experimental work already done, as well as upon the ideas of expert teachers. She is right, and her criticism calls attention to a weakness of the bibliographic notes in that they accord relatively little space to the results of experimentation.

Professor Shibli offers the following suggestions:

*First, the material in concrete geometry taught at present in grades 7 and 8 should be transferred to grades 5 and 6. Most of this material is too easy to be interesting and challenging to children of grades 7 and 8; and it is easy enough for grades 5 and 6.*

*Second, there should be developed a new course in geometry for the junior high school, which shall be neither wholly intuitive nor wholly demonstrative. The method of such a course should be inductive-deductive, intuitive-demonstrative. It should not consist of a long chain of reasoning; but of a collection of more or less isolated propositions, with a few short chains of reasoning and many original exercises. Such a course will show the pupil the meaning of demonstration, will increase his initiative, will satisfy his natural desire for intellectual mastery on the junior high school level, and will form a gradual and natural transition from intuitive to demonstrative geometry without mixing the two standards of reasoning in the same year (as would programs I, II, III).*

*Third, upon this foundation a suitable course in demonstrative geometry, not less logical and systematic than the present course, could be organized for the tenth year pupils who can profit by studying it. Such a course would be a basis for training in logical reasoning and a model for all life thinking.*

*Fourth, solid geometry should be given proper treatment in each stage.*

Mr. McCormack believes that we should keep the idea of transfer in mind throughout the course.

*We should take up applications with every group of theorems, not because these applications are valuable in life, but partly because they motivate the work by seeming valuable to the pupil and partly because they bring the geometry nearer to life and help the transfer to life situations. Then I think an occasional non-geometric application of reasoning to some life situation is valuable.*

*I would like to see a fairly large number of schools try consciously to carry over geometry to life situations by asking questions on non-geometric material and attempting to get the pupils to apply their geometric types of reasoning to these problems. Perhaps a good collection of life situations could be worked out to which geometric reasoning could be applied with a minimum of tacit assumptions. A question on College Entrance or Regents examinations on this sort of thing, at first optional and later required, would stimulate a more active attempt at transfer.*

Professor Upton writes: "As to experiments I consider Va (on transfer) *outstandingly most promising and worth while.* I know of nothing else that I should put ahead of it."

Miss Mary Kelly suggests that those who are planning experimental courses might arrange to publish enough details in *The Mathematics Teacher* so that others could try similar experiments.

The committee on geometry hopes that teachers of mathematics will be interested to study the synopses in Appendix A, to list for themselves the ideas that seem to them most worthy of trial, and to win the consent and cooperation of the proper administrative officers in their own school systems in devising experimental procedures for testing these ideas. This committee stands ready to do whatever it can to aid and abet such efforts.

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By vote of the Board of Directors on February 24, 1934 this committee is continued in office to act as clearing house for ideas concerning the teaching of geometry in secondary schools. It is hoped that voluntary cooperation will enable those who wish to avoid duplication of effort to learn of the work which others are doing and to profit by results already obtained. If this appeals to

you, kindly send to the chairman, Professor Ralph Beatley, Harvard Graduate School of Education, 19 Lawrence Hall, Kirkland Street, Cambridge, Massachusetts, a brief statement concerning experiments you are conducting or wish to conduct, as well as inquiries concerning promising fields for experiment. The committee has been granted authority to consult experts concerning statistical procedures which may come in question.

## APPENDIX A

### BIBLIOGRAPHICAL NOTES ON GEOMETRY

*Compiled by Miss Martha Hildebrandt, Mr. Albert J. Schwartz, and Mr. Ralph Beatley*

#### I. Periodicals, Yearbooks, Reports of Committees

Minnick, J. H. "The Recitation in Mathematics." *The Mathematics Teacher*, March, 1921.

The author warns against emphasizing the recitation form of class-room teaching at the expense of self-activity on the part of the pupil. "Ready made proofs of our geometry texts require little or no reasoning on the part of children. Nothing is of educational value which does not result in some form of activity on the part of the one being educated. Practical skill, modes of effective technique, can be intelligently, non-mechanically used, only when intelligence has played a part in their acquisition. If mathematical education is to be most useful, there must be intelligent mastery of facts. The rule of thumb method must be replaced by a rational method."

Webb, Harrison E. "The Future of Secondary Instruction in Geometry." *The Mathematics Teacher*, October, 1921.

The writer makes a plea for less formalism in our teaching of geometry. Referring to the large number of exercises in demonstrative geometry which contribute no factual knowledge, but serve only as drill material in formal logic, he says: "Exercises such as these have really prolonged the course in plane geometry and have altered the status of the subject from that of a classic to that of a bag of tricks."

The author's point of view may be discerned from the following paragraphs:

"Hilbert showed plainly enough the necessity of making Euclid 1:4 over into an axiom (assumption). Not to enter into the details of this question, it would appear equally logical and practical to assume that any two lines, or angles, or polygons, or curved figures, or solid figures which possess line symmetry are congruent. From this it easily follows that every case of congruence is reducible to symmetric relations, and a large number of geometric theorems follow almost directly."

"If geometry teachers could be persuaded to strain at a few less gnats and swallow a few more camels, the plane and solid geometry could be combined in one subject, for which one year's time would be ample. Once the idea is fully grasped that the chief aim of geometric teaching is to afford an analytical insight into the world of sense, the secondary curriculum will afford no better subject than a year of plane and solid geometry."

Allen, Fiske. "The Relative Emphasis upon Skill (Mechanical) and Applications of Elementary Mathematics." *The Mathematics Teacher*, December, 1921.

The author believes that there is little, if any, connection between ability to prove a theorem and the ability to apply it. He thinks that the type of mind which is interested in working out the logical demonstration of a theorem is not apt at that time to be interested in applying the theorem practically. He believes that in demonstrative geometry the stress should be upon the application of geometric reasoning to originals, but that the course in demonstrative geometry should be preceded by a separate course in inventional and problem solving geometry, in which many of the fundamental theorems may be taught without proof and used in applied problems.

Keyser, Cassius J. "The Human Worth of Rigorous Thinking." *The Mathematics Teacher*, January, 1922.

In this very fine short article Professor Keyser sets forth the need of mathematical discipline as an essential element of an appropriate education. He divides such an education into two parts: vocational and humanistic. The latter deals with such instincts, impulses, propensities and powers as are distinctly human. Among

the attributes characterizing the humanistic, a sense for logic and for rigorous thinking are important aspects.

He says: "It is plain that one of the great types of distinctly human activity—perhaps the greatest of all the types—is what is known as thinking. It consists in the handling of ideas as ideas—the forming of concepts, the combining of concepts into higher and higher ones, discerning the relations among concepts, embodying these relations in the form of judgments or propositions, ordering these propositions in the construction of doctrines regarding life and the world.

"And now what shall we say is the ideal of excellence that hovers above thought activity? What is the angel that woos our loyalty to what is best in that? What is the muse of life in the great art of thinking. . . . It is Rigor—Logical Rigor—which signifies a kind of silent music, the still harmony of ideas, the intellect's dawn of logical perfection.

"Can the dream be realized? . . . I am aware that these demands cannot be fully satisfied in mathematics, the logical science par excellence. Nevertheless I hold that, as the ideal of excellence in thinking, logical rigor is supremely important, not only in mathematical thinking, but in all thinking and especially in just those subjects where precision is least attainable. Why? Because without that ideal, thinking is without a just standard for self-criticism; it is without light upon its course; it is a wanderer like a vessel at sea without compass or star."

Kilpatrick, W. H. "The Next Step in Method." *The Mathematics Teacher*, January, 1922.

The author expands this theme: "The most widespread and imperative present tendency along methodological lines is the insistent demand that we get our students more fully 'into the game.' Modern education is increasingly seeing the need of treating pupils more as original centers of energy and of self-directed activity. This is not to deny an essential part in the education process to adult guidance and control. The problem of combining both essential factors in the highest effectual degree is yet to be solved. . . . For education to be its real self, the student must somehow from within feel the urge, and from this inner urge become—increasingly as wise guidance makes possible—a real center of self-directed thought and endeavor."

The author contrasts a passively receptive attitude of mind with a vigorous dynamic attitude, illustrating the various advancing degrees of activity, thus: "(1) A pupil memorizes the bare words of a demonstration; (2) a pupil memorizes the idea of a demonstration and can reproduce it in different words; (3) a pupil makes a given demonstration his own, it becomes his thought, he can use it in a new situation; (4) a pupil of himself demonstrates a proposition that has been proposed by another; (5) a pupil of himself sees in a situation the mathematical relations dominating it and of himself solves the problem he has thus abstracted from the gross situation."

The principal thought in the author's theme is that number (5) suggests the next step in method to be developed; that "as teachers, we are concerned not merely by the objective goals reached by the pupils, but quite as truly with the actual searchings themselves. The good teacher of mathematics nowadays knows, perhaps as do few others, that to have searched and found, leaves a pupil a different person from what he would be if he merely understands and accepts the results of others' search and formulation." The author believes that "the acceptance of this principle marks one of the definite advances made in our teaching of secondary mathematics within the past hundred years."

Schwartz, A. J. "The Teaching of Beginning Geometry." *The Mathematics Teacher*, May, 1922.

The writer presents the view that the learner in geometry should follow approximately the path of the historical development of the science. This, he affirms, can be done systematically and effectively by making a greater use of motion and symmetry than is done in the current textbooks.

The article describes a plan for a systematic approach to formal demonstrative geometry by means of an introductory chapter in geometric drawing. In this chapter, the elementary constructions are rationalized by a free use of motion and symmetry. The author believes that a rationalized approach, such as this, will provide a better introduction to demonstrative geometry than any amount of work of a purely empirical or mechanical nature.

Betz, William. "The Confusion of Objectives in Mathematics." *The Mathematics Teacher*, December, 1923.

Mr. Betz points out that two factors have been largely responsi-

ble in creating confusion in the objectives of secondary mathematics: the measurement movement in education and the enormous increase in the secondary school population. He shows that the attempt to deal fairly with "all the children of all the people" has tended to lower the standard for the whole school system. His solution is that the junior high school should furnish a common mathematical background for all pupils, but that an ability selection based on three factors, viz: (1) intelligence tests; (2) the complete school record of the pupil; (3) a comprehensive achievement test of the new type in at least two subjects, preferably English and mathematics, should be imposed on all those seeking to enter the higher courses in mathematics.

Mr. Betz insists that there is nothing undemocratic about such a plan. "It does not close the door to any worthy pupil, but it refuses to fit square pegs into round holes. Above all, it does not sacrifice standards of achievement. . . . It recognizes the fact that nearly all the sciences are becoming more quantitative from day to day, that practical men are among the first to deplore a weak school course in mathematics, that the traditional belief in the high cultural and disciplinary value of mathematics is not shaken."

Crafts, Lilian L. "Causes of Failure in Plane Geometry as Related to Mental Ability." *The Mathematics Teacher*, December, 1923.

The writer of this paper made a study of two groups of pupils described as follows. The first group consisted of sixty-two pupils who had failed in their first term's work in geometry; the second group was made up of forty-five pupils selected from three classes who were to go on with the second term's work.

The result of this study disclosed that the median score, measured by the Terman group test of mental ability, which the failing pupils attained was the score to be expected from pupils of their age and grade, while those passing had the score to be expected from seniors. This question is, therefore, pertinent: "Does it require the ability of a fourth year student to pursue successfully second year work?"

A detailed analysis of the study indicated that 47% of the failures were beyond the control of the schools, while 53% remain as a challenge to the efforts of parents, administrative officers, and

teachers. The writer makes the following suggestions as to how "some progress in reducing the number of failures might be made."

(1) Give an intelligence test to every pupil before he is assigned to classes.

(2) On a set of cards, have the pupils record the name, age, nationality of parents, previous success in mathematics, reason for any previous failure, and plans for the future.

(3) Eliminate from the classes those who seem incapable of the work.

(4) Stimulate the pupils to greater effort by the use of immediate incentives, such as exhibits of good work, honorable mention, etc.

In conclusion, she adds: "Further experimentation is necessary to devise means to find out which pupils are incapable."

Rorer, Jonathan T. "Present Tendencies in High School Mathematics." *The Mathematics Teacher*, January, 1924.

During the course of a discussion of the new requirements of the College Entrance Board (Document 108), referring in particular to the new one unit course, the writer makes this significant comment: "The whole reform movement which is based on ideas that have my entire sympathy faces the dangerous tendency of placing before the public more ideas than can be assimilated in the time allowed. Unless we are guarded, we shall increase the superficiality which our foreign critics claim to be characteristic of American education."

Hart, W. W. "Demonstrative Geometry." *The Mathematics Teacher*, March, 1924.

Mr. Hart's view of the chief function of demonstrative geometry as an instrument of general education may be discerned from the following quotations:

"Demonstrative geometry uniquely develops the habit of deductive thinking—more important than the habit of functional thinking. . . . There is reason to believe that the ideal and habit are transferred elsewhere. No other subject in the curriculum and no other part of elementary mathematics can fill the place of demonstrative geometry in developing this habit and ideal. Unless this function of demonstrative geometry is recognized as paramount, then demonstrative geometry loses its claim to usefulness in a scheme of general education."

Parker, Elsie. "Teaching Pupils the Conscious Use of a Technique of Thinking." *The Mathematics Teacher*, April, 1924.

The author's thesis is this: Granted that a transfer of training from one field of experience to another is possible under certain favorable conditions, the most effective method of realizing benefits therefrom is to teach pupils the conscious use of a technique of thinking. In developing this theme, she says: "The traditional method of instruction has been to let the pupil discover for himself a method of reasoning which he thereafter uses without in many cases being aware of the fact that he is using that mode of procedure." Putting her theory to the test, she describes a set of controlled experiments to answer this question: "Can pupils of geometry be taught to prove theorems more economically and effectively when trained to use consciously a technique of logical thinking; and furthermore, does such training, more than the usual method, increase the pupil's ability to analyze and see relationships in other non-geometrical situations?"

As a result of the series of experiments, which she recognizes is not conclusive because the experiments were not sufficiently extended, she reports the following conclusions:

"Original theorems in geometry were given to the classes before and after training in the conscious use of a technique of thinking. While the classes had the same number of correct proofs on the first one, on the second one after training, the experimental group had one and one-third as many complete and perfect proofs as the control, while, counting also the proofs which were on the right track, but not complete, the ratio was 12 to 21 in favor of the experimental group. As in the previous experiment, it was found that those trained in the new method exhibited more perseverance in endeavoring to find a proof. In this case, one and a half times as many of the control group quit before the end of the time allowed as in the experimental group. . . . These data would seem to offer conclusive evidence, in so far as one experiment can be considered to do so, that when pupils are taught to use consciously a technique of logical thinking, they try more varied methods of attack, reject erroneous suggestions more readily, and without becoming discouraged maintain an attitude of suspended judgment until the method has been shown to be correct.

"The data on the reasoning tests would seem to indicate that

such training in logical thinking with the materials of geometry tends to carry over these methods of attack and these attitudes to other problem situations not concerned with geometry."

Sanford, Vera. "A New Type Final Geometry Examination." *The Mathematics Teacher*, January, 1925.

In this article the author describes the weak points of the essay type of examination and discusses the desirability of constructing reliable new type examinations. Reliability is defined in this manner: If two tests rank the pupils in identically the same order, the reliability is said to be 1; if on the other hand, rank on one bears no relation to rank on the other, the reliability is 0.

In the course of her remarks, Miss Sanford describes a test devised in the Lincoln School. She says, "In its original form the test was in seven divisions, which may best be described by the type of question each contained. These were: Completion sentences; true-false conclusions from given data; true-false converse statements; matching reasons against conclusions; drawing valid conclusions from given data; computation; analyzing constructions. In order to get a measure of both the very good and the very poor pupils, it was planned that each part of the test should contain questions of graded difficulty; and to keep the test within a short time limit, it was proposed that the questions of each unit be arranged in order of difficulty. Ideally, then, a pupil's progress through each unit should be an indication of his ability and if he failed on the fourth question of a group, he might reasonably be expected to fail on all the later ones."

In commenting on the future work with tests she says: "One use of such tests . . . is, I believe, to give an idea of whether we are doing what we set out to do. We cannot be really doing it if in the end we find that children do not know fundamental axioms and definitions and that they cannot use theorems as tools. Tests, then, show some of the things we are not doing. The future work with the test depends in part on our being able to get a better picture of a child's attainment than can be gained from the judgment of his teacher or the essay examination. It appears to involve making a careful examination of the skills we believe geometry should develop, and then making a study of the best method of gauging these skills. . . These tests should be given at frequent intervals throughout the year. . . . The value of a final examina-

tion could then be measured by the ranking it gives a class in comparison with the composite of the year's tests and the part that each section of the final test plays in this analysis can be determined."

In conclusion, the writer adds this note of warning: "Such tests should not lead us to the point of believing that the sole object of geometry teaching is the achieving of norms or the raising of medians. In so far as tests help us to discover teaching difficulties and to remedy teaching mistakes, they are valuable."

Perry, Winona. "Student Difficulties in Geometry." *The Mathematics Teacher*, February, 1925.

In this article the author gives an analysis of a study of student difficulties in demonstrating original exercises in geometry. The study involved the judgments of many teachers as well as the efforts of several groups of pupils. The analysis of the study is expressed in the following "experimental outline":

A. Linguistic: Statements, elements, relations between elements.

B. Analysis of facts into their elements.
- I. Figures (basis of reasoning).
  - 1. Complexity, due to potency of words and phrases.
    - 1. As perpendiculars, bisectors, medians, producing sides.
    - 2. As types of figures.
  - 2. Unfamiliarity, due to inadequacy of connections.
    - 1. Relations between elements.
      - i. Number
      - ii. Obscurity of relations, as
        - a. overlapping figures,
        - b. construction lines.
    - 2. Number of non-available, habitual responses—as diagonal connecting two bisected angles; i.e. less recognition of partial identity of certain elements in the situation.
- II. Reasoning-activity
  - 1. Selection.
    - 1. Repeated responses (neglected relations or aspects of the figure).
    - 2. Accepted responses (from hypothesis, from possible conclusion).

2. Use in right relations.
    1. Number of essential steps—purposive thinking.
    2. Direction of the crucial step, or steps.
3. Drawing conclusions.
4. Judging conclusions—leading to bonds being strengthened from satisfaction.

Nunn, T. P. "The Sequence of Theorems in School Geometry." *The Mathematics Teacher*, October, 1925. (Reviewer B)

Asserts that teachers should make pupils realize clearly the logical structure of the geometrical system. Further, that the usual assumption concerning rigid motion be replaced by an assumption concerning the uniformity of space, affirming in effect that a geometric construction, wherever performed, produces the same figure. The object here is to avoid certain objectionable logical features inherent in the method of superposition (or in the language in which it is couched) and to show how proofs formerly depending on superposition can be reworded in terms of reconstruction.

Emphasizes that the idea of congruence does not account for all properties of geometric figures; that we need also the idea of parallelism, or its logical equivalent, similarity. He argues for a postulate of similarity in place of the parallel postulate, on the ground that young minds regard similarity as more important even than parallelism, and can appreciate at once the mutual independence of similarity and congruence (by comparing sphere and plane), whereas no example of the mutual independence of parallelism and congruence can be given short of non-Euclidean geometry.

In sum, he would replace the usual postulates concerning superposition and parallelism by two assumptions concerning congruence and similarity, as follows:

1) A given figure can be exactly reproduced anywhere.
2) A given figure can be reproduced anywhere on any (enlarged or diminished) scale.

*The Same.* (Reviewer C)

Professor Nunn proposes the following improvement in the order of propositions over the Euclidean order:

The whole of geometry would be organized on the basis of two (assumed) properties of space. Expressed in popular language they are:

1) A given figure can be reproduced (exactly) anywhere.
2) A given figure can be reproduced anywhere on any (enlarged or diminished) scale.

"No further property of space need be assumed, for no property of figures has been discovered which cannot be derived from one or both of these."

The first of these postulates displaces superposition in deriving propositions on congruent triangles, and the two combined displace Euclid's parallel postulate.

With this geometric architecture, he suggests, it is possible to proceed from the fundamental congruence theorems to the corresponding similarity theorems. Whether or not this is done, says the writer, is purely a question of expediency. Logical geometry, built up in this manner, would gain greatly in clearness and symmetry, says Dr. Nunn.

Strong, Theodore. "Teaching Geometry without a Text Book." *The Mathematics Teacher*, February, 1926.

The writer sets down the following three major difficulties which he avers every teacher of plane geometry has encountered:

1) "Introducing the class to the subject in such a way as to give its members enthusiasm as well as a good foundation.

2) "Adapting the text-book to his individual task . . . .

3) "Bringing the class to the point of attacking original exercises in a fashion truly efficient . . . ."

Too often, the author points out, in a beginning course in demonstrative geometry taught with the aid of a textbook the class, after a week or more, is swamped with mounting difficulties. He says: "The class has understood the new material, has accepted the definitions and admitted, in general, the truth of axioms and postulates; but that is about all. This material—the foundation stones of the geometrical edifice—is not in reality theirs to use. Their conceptions of essential elements of geometric figures are still vague. They cannot define these terms in accurate and concise language unless they have memorized and can repeat them parrot-fashion; and this is not knowledge. The axioms and postulates are still strange tools with whose use they are quite unfamiliar. . . . The result of this is that many pupils . . . begin to memorize the proofs of the theorems, with all of the unhappy consequences that ensue."

The writer's solution of this problem is: "Give from three to four weeks to the introduction, even in a course that devotes only one year to the subject. For this work the class, in the hands of an experienced teacher, needs no text-book and is better off without one." The writer believes that the pupils should be infused with the spirit of original investigation from the beginning. In this manner the minds of the pupils become dynamic instead of merely passively receptive.

Bowker, Elmer R. "Rigor versus Expediency in the Proof of Locus Originals." *The Mathematics Teacher*, February, 1927.

The writer of this paper believes that the schools should abandon "the rigorous method in the proof of locus originals. . . . The history of locus teaching in our schools (he says) is a record of consistent failure. We have not even succeeded in teaching the plotting of a simple locus, let alone the more difficult business of proving it after it is plotted."

He advocates the inclusion in our texts of seven proved or assumed theorems for reference. These theorems, he avers, would be sufficiently all embracing to prove any construction by simply quoting an authority. The proofs of these seven he would give in full, however, proving the theorem and its converse, or else the theorem and its opposite. Furthermore, he would show pupils why these pairs are logically equivalent.

"The time we spend in trying to teach the proof of locus problems could much better be spent in teaching our pupils to plot a locus successfully. The concept of locus which we are agreed is essential to a course in geometry can be gained by learning to plot a locus. The business of proving it adds little."

Taylor, E. H. "Mathematics in the Junior High School." *The Mathematics Teacher*, April, 1927.

The writer of this paper believes that an introductory course in geometry should be given to pupils in the 7th and 8th grades. "Intuitive geometry has a functional aspect through the unique training which it affords in the discovery and formulation of relationship . . . Intuitive geometry is absolutely essential as a preparation for effective work in demonstrative geometry."

The writer also believes that some training in demonstrative geometry should be given in the last half of the ninth year. He

says: "With a foundation of facts laid in the seventh and eighth years it is possible to give enough demonstrative geometry in a half year to be worth while." He would include in such a course only the simple theorems that can be proved by direct methods, and he would see to it that the *pupils* make most of the proofs. He affirms that "such a course can be made successful and that, based on it, a tenth grade class can complete the more difficult parts of plane geometry and solid geometry and have time left for some further study of algebra."

Evans, George W. "Postulates and Sequence in Euclid." *The Mathematics Teacher*, October, 1927. (Reviewer C.)

The author reaches the conclusion that there is little justification for distinguishing between axioms and postulates. After a thorough analysis of Euclid's Books, tracing the extent of dependence, he concludes that there is no sound reason for regarding geometry as a "fossilized science." "Even pedantic orthodoxy must see that the extensions already made, old as they are, were due to the broadening scope of the studies for which the elements were written as preparatory. Geometry, including algebra, is still preparatory. Mathematics beyond this stage still needs *elements*, but not necessarily the same elements that Archimedes needed, or even Legendre. Not the same lists from now on, and not the same order of succession."

*The Same.* (Reviewer B)

Analyzes the sequence of topics in Euclid; shows the independence of Euclid's Book V (on proportion); and suggests that we could do worse than to begin geometry with the subject of proportion, emphasizing "measurement numbers" from the start and dodging difficulty with incommensurables by the scientifically sound assumption that "two measurement numbers are equal if equal approximations can be found for them within whatever limits of precision may be arbitrarily prescribed."

Malsch, Dr. Fritz. "The Teaching of Mathematics in Germany since the War." *The Mathematics Teacher*, November, 1927.

The article gives a glimpse of the reform movement in Germany in the teaching of mathematics. This movement was greatly influenced by Felix Klein and his followers.

"The most significant of these new influences may be character-

ized by the catch words psychological approach, learning by doing, project method, and education for life.

"Instruction in geometry should afford a view of the systematic structure of the subject, and in proceeding from an empirical beginning to a greater and greater insistence on logical deduction should make the student realize the need for proofs and lead him from mere knowledge of geometric facts to a real understanding of the subject as a whole."

Sykes, Mabel. "Some Pedagogical Aspects of Geometry Teaching." *The Mathematics Teacher*, December, 1927.

For pedagogical purposes the subject matter of geometry should be organized by the teacher into teaching units. In each of these units, certain propositions should be stressed as basic, with other material grouped about them as supplementing certain definite notions. Thus, the topic of congruent triangles would be a unit in which the theorems for proving triangles congruent would be basic, while the theorems concerning the properties of the isosceles triangle, the equilateral triangle, etc. would be supplementary.

Ryan, James D. "Two Methods of Teaching Geometry—Syllabus vs. Text-Book." *The Mathematics Teacher*, January, 1928.

The author of this article describes some of the advantages of the syllabus method of teaching geometry over the text-book method. The principal advantage seems to be the result of substituting a live, active, alert, resourceful, adaptive, directed force for the inert, crystallized, expressionless pages of a book.

Beatley, Ralph. "Beginning Geometry and College Entrance." *The Mathematics Teacher*, January, 1928.

Discusses Document 108, published by the College Entrance Examination Board, which embodies a detailed statement of the revised requirements in plane and solid geometry

The College Entrance Board permits a choice between Plane Geometry (C) and Plane and Solid Geometry, minor requirement (cd). Commenting on the "pitifully small number" of candidates presenting themselves for examination in the minor requirement, Professor Beatley discusses the advisability or expediency of making the new requirement in plane and solid geometry the

sole requirement, or else withdrawing it for the time being. He concludes that neither of these methods of disposing of the vexing problem would be a satisfactory solution.

By way of a possible gradual improvement in the situation he suggests that the major requirement (C) be gradually enlarged in extent to include some parts of solid geometry, at the expense of some unstarred items in plane geometry—providing always that the emphasis on originals should not be reduced.

Moriarty, M. M. S. "Geometry Notes." *The Mathematics Teacher*, March, 1928.

The author discusses the proper and improper wording of various propositions found in text books. He favors simple direct statements of the facts described, wherever possible. The language used in dealing with the relations between angles and arcs in current text books receives the usual criticism.

Pratt, Gertrude V. "Popularizing Plane and Solid Geometry." *The Mathematics Teacher*, November, 1928.

Miss Pratt would popularize geometry, particularly for "the students who are spending a year studying a subject for which they, upon entering, have no burning interest" by such devices as historical references; mathematical recreations found in such books as "Mathematical Wrinkles" by S. I. Jones, "Mathematical Recreations" by Ball, and a "Scrap Book of Elementary Mathematics" by White; use of an attractive text book; resourceful interesting class-room procedure; discussion of interesting mathematical concepts, such as zero and infinity; correlation with solid geometry and algebra; apt discussions involving interesting physical and astronomical phenomena; cultivation of interest in geometric forms and designs; development of scrap-books on geometrical observations; field trips; use of logarithm tables in solid geometry.

Good, Warren R. and Chapman, Hope H. "The Teaching of Proportion in Plane Geometry" *The Mathematics Teacher*, December, 1928.

The authors of this contribution compare the treatments of proportion in several text books in geometry to indicate a definite

trend in the manner of presenting this material. This trend is shown to be toward a simple informal algebraic treatment, limited to the actual needs of geometric demonstrations.

The authors believe that further improvement in the teaching of proportion in geometry can be effected by a considerable reduction of the isolated presentation of the topic, and by a wider use in demonstrations of the axioms which form the basis of solving algebraic equations.

Christofferson, H. C. "A Different Beginning for Plane Geometry." *The Mathematics Teacher*, December, 1928.

The author suggests that the usual hypothetical construction that "every angle has a bisector," needed to prove the theorem relating to the angles opposite the equal sides of an isosceles triangle, be replaced by postulating the congruence theorem for which the hypothetical construction is assumed. The author believes this substitution would better serve psychological needs in beginning the subject.

Ziegler, D. G. "Concerning Orientation and Applications in Geometry." *The Mathematics Teacher*, February, 1929.

The writer discusses the importance of the attitude of the pupil toward the subject implied in the questions "What is geometry, anyway?" and "What good is it?" Mr. Ziegler believes that the answer to such questions as these should be sought by providing an intuitive beginning covering the following points, or topics.

1. Discussion of the methods of the ancients for restoring property lines.
2. The evident meaning of the word geometry.
3. Methods and difficulties of direct measurement.
4. Thales' introduction of triangulation, the indirect method of measurement.
5. Introduction of the idea of congruence, and its function in modern industry.
6. Introduction of the idea of symmetry.
7. Project: Search for an analysis of a geometric figure as found in modern industry and as used in life in general.
8. Transition from intuitive to formal work.

The following excerpt, based on experimental work undertaken, is significant. "From the standpoint of getting results and of the

time element, the six weeks' course in intuitive work was a success. It resulted in a decided growth on the part of the pupils in attitudes, appreciation and understanding. Concerning the time element, the six weeks spent in the beginning did not cramp the rest of the course for time. By the end of the course we were almost two weeks ahead of where we were the preceding year."

In the course of his remarks Mr. Ziegler makes this observation. "Perhaps the most important thing to look for in any course is the attitude that the pupils take towards their work It is important because attitudes of mind certainly have some bearing on how much and what kind of learning is going on."

Weissman, H. "Grouping in Geometry Classes." *The Mathematics Teacher*, February, 1929.

The author describes a plan used in the Franklin K. Lane High School, Brooklyn, N. Y., for classifying pupils in beginning geometry in three groups, according to ability. All three groups work in the same class room under the same teacher. Each group progroup proceeds according to syllabi differing both in quality and quantity of subject matter. The grouping, however, is not permanent for the whole term; pupils may at certain definite times be transferred from one group to another.

Although it can be easily surmised, says the author, that the inexperienced teachers were somewhat burdened in the process of teaching three classes in one, they all ended the term expressing their enthusiasm for the plan. There is no doubt that the teacher must be more alert to be able to take cognizance of the work of three different groups, but this is a matter of adaptation. The author thinks the advantages of the plan outweigh the disadvantages. Among the advantages of the plan, he lists the following.

1. The bright pupils were given full opportunity for individual development.
2. (The plan) facilitates the transfer of pupils from one group to another.
3. The C pupil is given a wider opportunity to come into contact with a wider knowledge of the subject.
4. There is greater emulation and also a spirit of self competition generated by the example set by the brighter pupils and the opportunities for promotion.
5. The teacher is made more conscious of the individual pupil.

6. The social setting in the tri-grouping of classes is nearer to real life conditions than the setting in which segregation of groups is practiced.

Greene, Harry A. and Lane, Ruth O. "The Validation of New Types of Plane Geometry Tests." *The Mathematics Teacher*, May, 1929.

"The new types of tests presented and described in this article are the result of an effort to go beyond the earlier conception of educational tests as mere survey instruments, to the more advanced notion of tests as specific guides to teachers in adjusting instruction to the needs of their individual pupils. These tests are designed to secure for the instructor reliable and economical measures for the information and the skills which contribute to successful achievement in plane geometry. The types of exercises used are: (1) Straight recall exercises, (2) completion exercises, (3) multiple response exercises, (4) bracketed phrases, (5) order of procedure, (6) wrong, useless and out of order statements in proofs." Of these six types, the first three are familiar to the reader of conventional tests; the last three are designed to test the pupil's ability to think through the steps of reflective thinking, and to determine whether or not a valid conclusion has been reached, in which, the authors aver, most objective tests fail.

Jackson, Dunham. "College Entrance Requirements in Geometry." *The Mathematics Teacher*, December, 1929. (Reviewer B.)

Those who believe in the existing courses in geometry at all will agree that the pupil ought to carry away with him an adequately comprehensive knowledge of geometric ideas, facts, and processes; an intimate acquaintance with the nature of deductive reasoning, as applied not only to detached items of argument, but also to the sustained building up of an extensive and coherent logical structure; familiarity with the independent use of deductive reasoning through the study of substantial "originals"; and some facility in the application of geometrical knowledge in the world of experience.

It is suggested that it may be found possible to preserve these essentials, with considerably more liberal recognition than has been customary hitherto of the principle that an elementary course need

not aim at the final articulation of *all* the facts that it embraces into a single logical framework. It may be possible to arrive at a readjustment of emphasis which will admit some of the important ideas of three-dimensional geometry in the first year, and at the same time bring geometry closer to the rest of mathematics and to the other sciences.

Durrell, Fletcher. "Value and Logic in Elementary Mathematics." *The Mathematics Teacher*, January, 1930.

In this article the author defends the thesis that a semi-formal treatment of geometry is most desirable in the high school. Discussing the prevailing method of presenting geometry in a critical vein, he says: "As this subject is ordinarily presented in our textbooks, pupils often complain that the first few theorems are obvious without these proofs and that these proofs at best seem like useless lumber. Often, moreover, such proofs contain elements of logical difficulty, require considerable mental effort for their mastery, and as courses of reasoning are not very convincing. As a result, at the very beginning of the study, in the minds of many pupils, a violent prejudice against the whole subject is formed.

"Pupils do not realize that the purpose of the first few demonstrations is not the discovery of new, unexpected, and valuable truths, but rather the simplification and systematic coordination of a number of rather obvious facts. Even if this purpose were pointed out to them, probably few would really understand what was meant by such a statement; and, if they did understand it, would not regard the aim as of sufficient importance to justify the expenditure of so much mental effort. The values of such a simplification and coordination are to them so vague and remote as not to seem worth while, and there is much in their view which is essentially sound.

"It is possible for a teacher to take a textbook written on the present basis, and teach it in a value-logic way. Thus at first the early theorems may be treated and established in an informal way, and later, near the end of the year may be studied in the usual formal demonstrative manner. Many teachers are today following this plan.

"Euclid's treatment of geometry was written for adults or for mature students. Until a recent date in our own country the subject was taught only in colleges. In time, as new subjects were

added to the college courses of study and the secondary school system developed, the study of geometry was gradually pushed down into the academy and high school. But when they transferred it, it was not modified to meet the new situation, and especially the more immature type of pupil. It was kept in its original abstract and highly technical form.

"Moreover, Euclid had in mind to build up a pure logical system of geometry on an apparently minimal foundation. But this presentation of such a set of fundamental concepts and principles has not been entirely successful. Many attempts have been made to improve on Euclid's system of first principles, but no general agreement exists on the matter. The problem of finding a pure, *a priori* logical theory of geometry has not been solved in a generally accepted way. It is a question whether such a solution is possible. If this is the case, a pure logical theory going to the utmost roots of the subject seems impossible. Hence, in all cases, we shall merely have a question as to the degree to which useful concrete experience shall precede and be mixed with abstract studies. This would leave us free in the high school curriculum to select that point on the scale best suited to secondary psychology and other conditions."

That this movement has gained some headway, even in the colleges, the author reveals by quoting from the preface of the "Survey Course in Mathematics" by N. J. Lennes, written for the freshman class in college, viz.: "There is a considerable direct appeal to intuition where the severely logical path is not easily trod by the student."

In summing up, the author makes this significant remark: "The method of giving the principles of function and value the leading role and making formal technique secondary and auxiliary is one which should be used in many other, and perhaps all branches of study, and in life in general in all its details and relations."

Evans, George W. "Proposed Syllabus in Plane and Solid Geometry." *The Mathematics Teacher*, February, 1930.

The author proposes a "list of propositions, 104 in number, for plane and solid geometry, the same total content as the list of 181 that constitutes the College Entrance Examination Board's definition of those two subjects. It is based upon measurement, and uses systematic approximation as a means of dealing with irra-

tionals; also algebra and trigonometry are used wherever they contribute to clearness and brevity.

"The propositions are divided into successive groups so that in each the student may realize that there is an immediate aim for the group, as well as a general aim toward which each group makes obvious progress.

"The assumptions listed in the beginning are those usually made, whether tacitly or not, in current American textbooks."

Gugle, Marie. "Geometry in the Junior High School." *The Mathematics Teacher*, April, 1930.

In discussing the work of the ninth grade, the author says: "The work of the ninth grade will be more largely algebraic, but constantly illuminated by geometry and graphs and continuing arithmetical computation in checking. If in the junior high school the pupil is to survey the whole field of elementary mathematics, he should have a glimpse into numerical trigonometry and into demonstrative geometry. The latter might well be only enough to show the pupil what a demonstration is, in order that he may know whether or not he wishes to elect the subject in the senior high school."

Taylor, E. H. "The Introduction of Demonstrative Geometry," *The Mathematics Teacher*, April, 1930.

Professor Taylor reviews the recommendations of the Report of the Committee of Ten on Secondary Studies (March, 1893), the Report of the National Committee of Fifteen on Geometry Syllabus (July, 1912), and the Report of the National Committee on Mathematical Requirements, and cites the practices in other countries to reinforce his arguments that preparatory work in geometry in the grades or junior high school is vitally needed to prepare for the demonstrative geometry in the high school.

He says: "We teach demonstrative geometry so that high school pupils may learn the methods of geometric proof. The most significant cause of early discouragement and later failures is a lack of adequate introduction to geometric notions through concrete experience and intuition before beginning formal proof."

Stone, John C. "A One Year Course in Plane and Solid Geometry." *The Mathematics Teacher*, April, 1930.

The author takes the position that it is entirely feasible to re-

construct the course in tenth year geometry so as to include some geometry of space. He suggests that the ratio be in the neighborhood of a 3:1 ratio, but thinks that "considering the immaturity of the child in the tenth grade, both the logic and the simplicity of the subject can best be attained by devoting about two-thirds of the year to plane, followed by solid, bringing out the one-to-one correspondence of the theorems of solid to those of plane when such occurs." The justification of his position is contained, for the most part, in the following excerpts:

"Unlike algebra, geometry is not to any great extent a preparatory subject. Only a small part of geometry is needed to understand the mathematics that follows. And as for the preparatory feature, there is nearly as great a need of solid as of plane in future mathematics. From the standpoint of practical uses of the subject, no strong brief can be made for either plane or solid. Most of the everyday uses of the subject are now taught in the intuitive geometry of the junior high school, formerly called mensuration in our arithmetic."

Longley, W. R. "Geometry as Preparation for College." *The Mathematics Teacher*, April, 1930.

The significant points of this article are encompassed in the following paragraphs:

"The instructors with whom I have talked agree in general that their students possess a knowledge of geometric ideas which is sufficient for the college courses in mathematics. The instructors usually add that only a small amount of such knowledge is really essential and point out a few instances where ignorance of the simplest facts of solid geometry is surprising. But there has been no complaint about deficiency of knowledge of the facts of plane geometry and I believe we are justified in assuming that the preparation in this phase of the subject is quite satisfactory."

Not so, however, he adds by way of digression, with regard to algebra: "Algebra is the most fundamental preparatory subject for college mathematics, and college teachers have always complained about the training which their students have had. It may not be out of place to remark, parenthetically, that the results of the efforts of the complaining college teachers are criticised just as severely by the professors in various departments of science and

in the engineering schools. I believe that the fault does not lie primarily in our teaching of algebra. The difficulty is in carrying over the knowledge there gained to later work. They have too little facility in the world of experience, that world being largely later courses in mathematics and science.

"The hopelessness of developing by this method any reasonable degree of efficiency in finding the right mathematical tool when wanted has led to the attempt to develop courses in general mathematics. I know very little how such courses have worked in secondary schools. Somewhat similar attempts in college mathematics have not been very successful. It appears that only one new thing can be taught at a time and each new thing must be fairly closely related to what has preceded it. In my opinion a better remedy than general mathematics is cumulative mathematics.

"The possibility of integrating to some extent the separate courses in mathematics applies to the present courses in plane geometry for one year, and, for diminishing numbers of pupils, in solid geometry for a half year, but more particularly to the new course, which consists of a year's work in geometry, plane and solid, to be followed in some cases, by a half year's work in advanced geometry. In constructing the new course we start with a comparatively new slate and it will be easier to weave in a judicious mixture of algebraic problems than it would be to introduce them into our present courses so hallowed by the tradition of Euclid."

Stokes, C. N. "Building Tests in Junior High School Mathematics." *The Mathematics Teacher*, November, 1930.

Mr. Stokes says: "The fundamental problem in teaching junior high school mathematics resolves itself into a matter of effecting real learning. The pupil must have a mastery of the minimum essentials. He must have those skills and understandings which are deemed the necessary equipment of every normal individual who must make the necessary adaptations to the environmental conditions of the well ordered life. He must have the ability which stands the test of normal and habitual use under practical conditions in the everyday life situations. Training in junior high school mathematics must add materially to the demands of society.

"One consideration in the process through which the learning may be acquired is to determine the effectiveness of instruction. A

diagnosis must be made periodically in order that any deficiencies in learning may be located and corrected.

"The modern trend is away from the commercial standardizations toward the new-type examinations which more nearly fit the materials as they are presented in the respective school organizations. The criteria set up for the construction of tests are:

1. Every test should attempt to measure a pupil's ability to master the subject-matter that has been presented to him.

2. Every test should emphasize primarily those parts of the subject-matter which are fundamental and to which the pupils have directed the most attention.

3. The scoring of each test should be arranged so that in scoring them all teachers may obtain exactly the same results.

4. Every test should be reliable; that is, if two tests were arranged, the pupils taking them should attain the same ranking.

5. Every test should be so constructed that it is possible to set some form of standard of achievement for a pupil."

In constructing a test, says the author: "The first step is an analysis of the course to identify the specific abilities which constitute the minimum essentials to be emphasized in the teaching. The second step is to draft a set of questions which are objective, comprehensive, reliable, and economical as to administration and scoring."

The paper contains a sample mastery test for a unit in demonstrative geometry for the ninth grade.

Orleans, Joseph B. "The Fusion of Plane and Solid Geometry." *The Mathematics Teacher*, March, 1931.

The author makes some pertinent suggestions on two vital points relating to the fusion of plane and solid geometry, viz.:

"If a school system were *definitely and truly organized on the 6–3–3 basis and the mathematics of the seventh, eighth, and ninth years included intuitive geometry with possibly a unit of demonstrative geometry*, then it would be possible to devote one-third of the tenth year to solid geometry as outlined in the National Report.

"It should be possible to omit some of the plane geometry without weakening the course materially, in order to make room for enough of the solid geometry that is considered worth while.

"Where can time be saved from the plane geometry for the new work? The answer is to postulate more than we do. Do not have

the pupils spend unnecessary time on book propositions the proofs of which depend on auxiliary lines that they themselves cannot suggest. I do not mean to omit them altogether. After these proofs have been developed and demonstrated, their importance established and the facts learned, they should be applied in exercises, but their proof need not be repeated just because they are numbered propositions. This would not weaken the logical background."

Beatley, Ralph. "Notes on the First Year of Demonstrative Geometry in Secondary Schools." *The Mathematics Teacher*, April, 1931.

As a possible solution of the more or less controversial question—What to do with plane and solid geometry?—Professor Beatley suggests that geometry and algebra be taught in parallel courses in the tenth and eleventh grades, giving the geometry Monday, Tuesday, and Wednesday of each week in Grade X, and twice a week in Grade XI.

He says: "Probably not more than one-tenth of all the pupils in the United States who take a first year of demonstrative geometry go on to the study of solid geometry. Even in the strongest schools this fraction rarely exceeds one-sixth. It seems clear that some study of the geometry of three dimensions has value for almost all pupils of secondary school age, but that few of them will have this advantage unless we make definite provision for it either in the informal geometry of the junior high school years, or in the first year of demonstrative geometry. I see good reason to include it in both, putting as much as possible of the three dimensional work into the informal geometry of the junior high school, and fitting the remainder into the first year of demonstrative geometry." In furthering this plan he proposes "the elimination of thirty or more theorems from the plane geometry syllabus, one-third to be dropped outright and the other two-thirds to be kept as definitions or assumptions and used in the support of original exercises." From solid geometry he proposes the addition twenty-eight theorems, for only seven of which the student would be asked to give formal proofs.

The article includes a proposed syllabus, a striking feature of which is the inclusion among the assumptions of many obvious

theorems whose proofs would seem useless or wholly unnecessary to beginners.

Jackson, D., et al. "Report of the Committee on Geometry." *The Mathematics Teacher*, May, 1931.

The point of view of the committee is disclosed in the opening paragraph of its report, viz.:

"There is a wide-spread conviction held by a group of teachers impressive both as to numbers and as to scientific and educational standing, that the proposed rearrangement will be a fundamental improvement in the teaching of geometry. The leaders of this group are teachers who believe in the importance of mathematics as part of a liberal education. Their desire is not to diminish the amount of time given to the teaching of geometry, but to increase the effectiveness of that teaching. Their chief concern is to remedy the situation created by the progressive disappearance of solid geometry from the course of study of the average high school pupil, and to bring some important parts of it into the program of pupils who under present conditions would learn nothing of it at all."

Austin, M., et al. "Report of the Second Committee on Geometry." *The Mathematics Teacher*, October, 1931.

As a result of its deliberations, the committee presents two tentative courses combining plane and solid geometry. A brief description of the courses is contained in the following paragraphs:

The first course is called a tandem course because the material from solid geometry is introduced separately after the plane geometry is finished. In this course proofs are required for forty-six theorems of plane geometry and twenty-eight of solid.

The second course is called a fused course. In this course, the solid geometry concepts are introduced at appropriate places by imaginative questions. Rigid proofs are required only for the plane geometry theorems. In this course thirty-eight theorems and eighteen corollaries of plane geometry are proved. Besides the imaginative questions of solid geometry, proofs for fifteen theorems are required. Most of the formulas are assumed or developed intuitively. Many numerical exercises depending on these formulas would be included to complete the course.

The article includes syllabi for the two courses.

Garabedian, C. A. "Some Simple Logical Notions Encountered in Elementary Mathematics." *The Mathematics Teacher*, October, 1931.

Explains why proving the proposition "If $B$, then $A$" and its converse "If $A$, then $B$" establishes $A$ as a necessary and sufficient condition for $B$: i.e. $B$ is true *if and only if* $A$ is true.

Lee, J. Murray, and Lee, Dorris May. "The Construction and Validation of a Test of Geometric Aptitude." *The Mathematics Teacher*, April, 1932.

The authors constructed a prognostic test in geometry which was given to about 600 students. The validity of the test was later determined by a comparison with achievement tests at the close of the course. Only those items in the test which showed a high degree of correlation were retained in the calculations. The correlation of the selected tests with the achievement test was .720 $\pm$.28. As an indication of the reliability of the test for prognostic purposes, it is interesting to note the correlation of the achievement test with the semester marks. This correlation was .789. Hence the prognostic test for determining the aptitudes of the pupils is about as reliable as the achievement test.

Hedrick, E. R. "What Mathematics Means to the World." *The Mathematics Teacher*, May, 1932.

The point of view of the author is revealed in the following excerpts:

"The quality that persists in mathematics is not logic. The meaning of mathematics to the world is not its axiomatic scheme of deduction. We have, in fact, only been forced to that by the awkwardness of persistent contradictions that have arisen throughout mathematical history. I recommend that you study the real origin of geometric knowledge; that you study the furious controversies that surround the discovery and introduction of the calculus, the first forms of which (and some of the present teaching) are decidedly illogical. Geometry is to many minds again a branch of logic. As a school subject, it is attacked always on the assumption that it is nothing more than a branch of logic. The logical foundation of geometry will always be a monument to man's mental powers. But it means more than that to the world. The control of form, as in a modern bridge, in a modern skyscraper, in the beautiful architectural dreams of ancient and modern days, in

the vast scheme of heavenly bodies, in the flow of fluids, of late so magnified in importance through streamlining of airplanes and automobiles—such is what geometry means to the world of form, and the control of form, and all that depends on it."

Durell, Fletcher and Durell, Thomas. "Getting the Most out of Objective Tests in Mathematics." *The Mathematics Teacher*, November, 1932.

In this article, the authors present various ideas on how to get the most out of the new type tests as instruments of instruction. To give an idea of the character of these comments, we quote the following, referring to the multiple choice type:

"If the multiple choice test is fully developed, the values which it shares with the completion and true-false types may be greatly increased. It is customary in the list of possible answers to have only one correct result. The disadvantage of this is that if the pupil knows this to be the case, as soon as he has come to a correct answer the remaining answers in the list have no further interest for him. A much better method is to include in the list of alternatives two or even more correct answers. Obviously, when the pupil knows that there may be more than one correct answer, he will be on the alert till he has tried the last item in the list, and the solution of the problems will become a many-sided game."

Moore, E. H. *On the Foundations of Mathematics*. 1902. Reprinted (1926) in the First Yearbook, National Council of Teachers of Mathematics.

Discusses the foundations of geometry as set forth by Hilbert and by Veblen. Refers to the efforts of John Perry in England to weaken the authority of Euclid in geometry, at least with respect to the sequence of propositions, and in favor of familiarizing the student by observation and experiment with the ideas of geometry before demanding a logical consideration of these ideas. "It is scientifically legitimate to take a large body of basal principles instead of a small body."

Reeve, W. D. *The Teaching of Geometry*. Fifth Yearbook, National Council of Teachers of Mathematics, 1930.

This is the introductory article in the Fifth Yearbook. It includes a discussion of "Informal Geometry," i.e. content of the course and reasons for early introduction of geometry as well as of

'demonstrative geometry,' differentiates between the two and states the value of the first to the second.

"The purpose of (demonstrative) geometry is to make clear the meaning of demonstration, the meaning of mathematical precision, and the pleasure of discovering absolute truth."

"Informal geometry represents about all the geometry that many pupils are capable of understanding."—"The pupil with fairly high intelligence quotient will derive more pleasure and interest from doing than from merely depending upon his intuition; he will obtain satisfaction in demonstrating that his intuition is correct."

He suggests some omissions and additions and includes a paragraph on criticisms of and suggestions for geometry and textbooks. He briefly discusses the inclusion of some solid geometry with the plane, also of algebra in geometry proofs where failure to do so would make the work too difficult.

Under "foundations of geometry" he includes the nature and purpose of definitions, how precise they should be, distinguishes between theorems and problems and lists the basic propositions.

"A good plan for teachers to follow is to choose a carefully written modern textbook in geometry, follow the sequence given there as carefully as possible, using all the ingenuity they can, emphasize the work on original exercises, and cultivate the originality and imagination of the pupils as much as possible."

The author lists the four methods of proofs of original exercises as "synthetic method," "analytic method," "method of loci," and "indirect method," and gives the pupil directions for proving propositions. He states that there is much misuse of blackboards in this country and suggests that neat and accurate drawings on large pieces of cardboard are more economical of time when complicated figures are necessary. Proofs of propositions in texts are models for exercises.

Longley, W. R. *What Shall We Teach in Geometry?* Fifth Yearbook, National Council of Teachers of Mathematics, 1930.

The number of students who need mathematics for engineering, physics, astronomy, chemistry, mathematical statistics in the field of economics, business, education and the natural sciences, in medical research, etc. is increasing rapidly and the amount of mathematics required in each field of application has multiplied many fold. Geometry is valuable because (1) of its logical exposi-

tion, (2) a knowledge of geometric facts and relationships is as desirable in social life as the ability to recognize the most common quotations from Shakespeare, (3) the necessity and utility of its mensuration and formulas and methods, and (4) the cultivation of space perception. The traditional course in solid geometry is considered futile by many teachers as well as students and so arrests progress in the mathematical ladder. Two solutions are offered by the College Entrance Board. (1) Omit formal proof of 36 propositions in Book VI but content of the book must be understood and followed up by many simple problems making use of them; the introduction of numerical trigonometry of the oblique triangle will liberate us from too great dependence upon the 30–60 right triangle. (2) Mathematics *cd*, a combination of one year of plane and solid geometry for which the author suggests a satisfactory course and textbook will have to be developed.

Sanford, Vera. *Demonstrative Geometry in the Seventh and Eighth Grades*. Fifth Yearbook, National Council of Teachers of Mathematics, 1930.

The author suggests that demonstrative geometry can be introduced informally in the seventh and eighth grades with the brighter pupils as an integral part of their study of geometric relationships. She describes an experiment in which the three steps of procedure are trial and error leading to the formation of a rule of thumb, tests of the rule, and reasoning.

There is included in this work a discussion of unproved converses, phrasing the opposite of a given statement, the arbitrary nature of a definition, the idea that a postulate is acceptable without proof, and the contrast between a direct proof and a proof by exclusion. From her experience she believes that this procedure makes for a greater unity in the geometry of the junior high school, provides opportunity for individual work of high quality, and bridges the gap between informal and demonstrative geometry.

Orleans, Joseph B. *A Unit of Demonstrative Geometry for the Ninth Year*. Fifth Yearbook, National Council of Teachers of Mathematics, 1930.

An outline of a unit of demonstrative geometry to cover from six to eight weeks that can be fitted into the ninth year if algebra is begun in the eighth year and a foundation of intuitive geometry is laid in the seventh year.

It includes a discussion of preliminary definitions, and exercises on vertical angles, congruence theorems, isosceles triangle theorems, parallel lines, and similar triangles.

Seidlin, Joseph. *A Unit of Demonstrative Geometry for the Ninth Year.* Fifth Yearbook, National Council of Teachers of Mathematics, 1930.

An outline of a unit "to show the pupil what 'demonstration' means," whereby he hopes to "leave with the pupil a pleasant and lasting impression that there is a subtle, mentally satisfying quality in 'proof by reasoning' not found in 'proof by experiment' or in 'intuition'."

Includes some work on vertical angles, congruence, inequalities, parallel lines, sum of angles in a triangle and polygon, parallelograms, and areas, including an informal treatment of the Pythagorean Theorem.

Wilt, May L. *Teaching Plane and Solid Geometry Simultaneously.* Fifth Yearbook, National Council of Teachers of Mathematics, 1930.

The author suggests that more and more solid geometry be absorbed into plane geometry; that as a result the teaching of geometry will become more dynamic and the understanding of it less difficult for most pupils.

The author contends that definitions might be enlarged to include space as well as plane relationships. For instance, in the study of angles, include dihedral and polyhedral as well as plane angles; use spherical as well as plane triangles; combine work on parallel planes with work on parallel lines; parallelepipeds with parallelograms; volumes with areas and spheres with circles. The treatment of loci would be enriched by using both two and three dimensional space. This combination would eliminate about one-third of the content of the present course in plane geometry.

Allen, Gertrude E. *An Experiment in Redistribution of Material for High School Geometry.* Fifth Yearbook, National Council of Teachers of Mathematics, 1930.

The material for the tenth school year for which elementary algebra is a prerequisite is regrouped under five headings: 1) congruence and equality, which also includes the sum of the angles of triangle and polygon and the theorem of Pythagoras, 2) similarity

and symmetry, 3) form and position, including work with rectilinear figures and solids bounded by planes; circles; and locus both in space and plane and informal treatment of variables and constants, 4) geometric constructions more or less complex applying previously acquired knowledge to new situations, 5) mensuration, i.e. "development of standard formulas for area and volumes" and "numerical computations involving these formulas applied in miscellaneous problems of practical value." "The teachers who have cooperated in the experiment are unanimous in recommending a more fundamental basis for organizing elementary and advanced geometry than a division into plane and solid geometry; an increased number of postulates and a decreased number of theorems to be proved in the elementary tenth year course; close correlation of plane and solid geometry, trigonometry, algebra, and arithmetic; significant originals and applied problems; organization of subject matter into a coherent whole made up of self-consistent interdependent units."

Birkhoff, George D. and Beatley, Ralph. *A New Approach to Elementary Geometry*. Fifth Yearbook, National Council of Teachers of Mathematics, 1930.

The authors discuss the purposes of geometry and the advantages and disadvantages of the Euclidean and Riemannian methods of approach to geometry. Then they "formulate a method of approach which may eliminate most of these disadvantages and at the same time embody the fundamental advantages of both methods." They take for granted the notion of number, and the self-evident fact of linear and angular measurement and scale drawing. With five fundamental principles—including one principle of similarity—they can prove six basic theorems, among which are two other principles of similarity, the theorem concerning the sum of the angles of a triangle, and the Pythagorean Theorem, and six corollaries. From these they say "all theorems in geometry can be derived easily and naturally." Congruence is considered a special case of similarity and "the treatment of parallels can be derived from the principle of similarity."

As advantages of this approach they say: "In the first place it is severely logical. In this respect it offers training as satisfactory as that of Euclid, and is much more direct. It lends itself admirably to explicit consideration of the place of undefined terms,

definitions, and assumptions in any chain of logical reasoning and leads to the development of those same habits, attitudes, and appreciations which all teachers of geometry claim for their subject. In the second place, it takes advantage of the knowledge of number and of linear and angular mensuration which the student possesses, and so does away with the feeling of artificiality which is inevitable when seemingly self-evident propositions are 'proved'. In the third place, it leads very naturally to the elementary facts of analytic geometry and makes it apparent in this way that geometry is really a self-consistent discipline whether or not such things as points, lines, and planes really exist."

Swenson, John A. *Graphic Methods of Teaching Congruence in Geometry.* Fifth Yearbook, National Council of Teachers of Mathematics, 1930.

"The mathematician does not consider the method of proof by superposition very satisfactory. The psychologist says we should introduce no unnecessary habits in connection with the processes of learning."

The author discusses the teaching of congruence by graphic methods, arguing that algebra and geometry should be used to reinforce each other as mathematics is fundamentally a study in variation. He verifies the first congruence theorem by the actual cutting out and placing of one triangle on the other. Then he explains what he means by variation in the same sense, and by graphing shows that 1) given a side and an adjacent angle of a triangle, the other adjacent angle and its opposite side vary in the same sense through the whole range of possible values; and 2) if two sides of a triangle remain constant, the included angle and the third side vary in the same sense. The first of these principles he uses to prove two triangles congruent if they have two angles and the included side of one respectively equal and so forth, and the second to prove two triangles congruent if three sides of one are respectively equal to three sides of the other.

Upton, Clifford Brewster. *The Use of Indirect Proof in Geometry and Life.* Fifth Yearbook, National Council of Teachers of Mathematics, 1930.

The author states that the great purpose in teaching geometry is to show the pupil how facts are proved, the nature of a deductive

proof, and to furnish pupils with a model for all their life thinking. He shows that indirect proof is not popular with many pupils and teachers due to the fact that it is not well understood because of too early introduction in the course and insufficient practice in using this tool. Indirect proof is a form of analysis.

He discusses the nature of an indirect proof, the method of elimination, and the law of converses, summarizes the fundamental principles of logic on which it is based, and gives detailed applications of indirect proof to geometric propositions as well as to this type of reasoning in life.

Schlauch, W. S. *The Analytic Method in the Teaching of Geometry.* Fifth Yearbook, National Council of Teachers of Mathematics, 1930.

"Whatever is done in rearranging the subject matter of geometry, we must hold fast to the analytic method of attack if we would preserve its value as a training in logic and in original thinking, and inspire the confidence which lies back of the bravery necessary to attack its difficulties. Analysis is the method of discovery and the only method of organizing the subject matter of geometry which gives sufficient command of the logical processes to justify its study."

The author discusses the value of geometric training, pointing out that the analytic method is the heart of geometric work. He gives three illustrative examples of the method, an original theorem, a construction problem, and a problem of computation. He states that the student needs training in analysis applied to the various types of geometric subject matter found in the conventional five books of plane geometry and that a year of consecutive work produces the best results. If an introduction to demonstrative geometry is given in a junior high school in which the analytic method has been used, the remainder of the plane geometry and the solid geometry can be covered successfully in one year of senior high school work. But if one must begin the plane geometry in senior high school and teach both plane and solid geometry in one year, the student's view of both is inadequate.

Young, J. W. *Symmetry.* Fifth Yearbook, National Council of Teachers of Mathematics, 1930.

The author suggests that since the idea of symmetry abounds

in nature and art, the child already has it when he approaches his first study of geometry even though he does not know it by that term. This is the reason European schools have for some time made use of this idea in the introductory work in geometry and "it may be expected that (in this country) the work in intuitive geometry especially can be benefited by a use of symmetry."

The author defines axial symmetry and from his definition immediately leads to certain results. He also shows how the construction of the perpendicular bisector of a line segment and of the bisector of an angle may be justified by considerations of symmetry.

Betz, William. *The Transfer of Training with Particular Reference to Geometry*. Fifth Yearbook, National Council of Teachers of Mathematics, 1930.

"The problem of transfer of training is fundamentally one of good teaching. But whereas the 'born' teacher may obtain excellent results by an instinctive application of the principles on which transfer depends, there is no doubt that in the average classroom the efficiency of the work could be greatly increased by a conscious cultivation of these cardinal principles. The following findings which have at last emerged from the mental discipline controversy might well be incorporated in the creed and the daily practice of every progressive teacher:

1. Training for transfer is a worth-while aim of instruction; from the standpoint of life it is the most important aim.
2. Transfer is not automatic. We reap no more than we sow.
3. Every type of specific training, if it is to rise above a purely mechanical level, should be used as a vehicle for generalized experience.
4. The cultivation of thinking is the central concern of education."

Mensenkamp, L. E. *Some Desirable Characteristics in a Modern Plane Geometry Text*. Fifth Yearbook, National Council of Teachers of Mathematics, 1930.

The author discusses the history of geometry, showing how it was originally a subject for adults but how there has been a downward trend and a corresponding revision in material to make it acceptable to immature learners. He shows how more than twenty

years ago Felix Klein urged 1) that the instruction should emphasize the psychologic point of view which considers not only the subject matter but the pupil, and insists upon a very concrete presentation in the first stages of instruction followed by a gradual introduction of the logical element; 2) that there should be a better selection of material from the viewpoint of instruction as a whole; 3) that there should be a closer alignment with practical applications; 4) that fusion of plane and solid geometry, and of arithmetic and geometry should be encouraged. Klein adds that the final stage in carrying out a reform is reached when the recommended changes and improvements are embodied in textbooks for classroom use and suggests a guide for bearing these facts in mind. His outline embodies 1) points pertaining to the book as a whole, such as recency of the edition, educational experience and scholarship of the authors, its conformity with requirements, mathematical point of view and presentation of material; and 2) points pertaining to specific phases of the book such as definitions, constructions, theorems, and exercises.

(*To be Continued*)

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# Abstracts of Recent Articles on Mathematical Topics in Other Periodicals*

*By* NATHAN LAZAR

*Alexander Hamilton High School, Brooklyn, New York*

### Algebra

1. Chistakoff, I. *Notes on progressions.* Matematika i fisika v srednei shkole. (Mathematics and Physics in the secondary schools.) Quarterly Publication of the Commissariat of Education and Public Instruction of U.S.S.R. 1: 15–17. 1934 no. 1.

A generalization of certain properties of arithmetical and geometric progressions.

2. Cohen, Jacob. *Algebraic aids in arithmetic.* High Points. February 1935, pp. 39–42, vol. 17, no. 2.

A discussion of various applications that the teacher of algebra may exhibit to make arithmetical manipulation more interesting, more useful, and more meaningful. As a result of his understanding of the arithmetical operations, which he was once taught to perform mechanically, the pupil gets a sense of power that increases his respect for algebra.

3. Emch, Arnold. *New models for the solution of quadratic and cubic equations.* National Mathematics Magazine. 9: 162–64. March 1935.

The models described demand, for a complete understanding of the method of solution, a knowledge of solid analytic geometry. Although of great theoretical interest, they can therefore be of but little pedagogic value in the usual course in advanced algebra, given in the secondary school or at college.

4. Hartung, M. L. *Wisconsin algebra tests.* Wisconsin Journal of Education. 67: 216–17. January 1935.

A discussion and analysis of some of the common errors in the answers to a test given in 1933 and 1934 to several thousand pupils in Wisconsin. A detailed report of the errors on 500 papers chosen at random is available in mimeographed form and may be obtained by writing to Prof. T. L. Torgerson at the University of Wisconsin.

5. Laritcheff, P. (a) *Systems of equations of the first degree.* Matematika i fisika v srednei shkole. (Mathematics and Physics in the secondary schools.) Quarterly Publication of the Commissariat of Education and Public Instruction of U.S.S.R. 1: 61–66. 1934 no. 1.

A discussion of the methods of teaching the solution of sets of equations in two and three unknowns. The writer urges that the problems given the pupils should be such as might arise in concrete situations, preferably out of the current life in Russia.

(b) *Quadratic equations.* Matematika i fisika v srednei shkole. (Mathematics and Physics in the secondary schools.) Quarterly Publication of the Commissariat of Education and

* The summaries of the articles in Russian would have been impossible without the kind aid of Mr. Aaron Bakst.

Public Instruction of U.S.S.R. 1: 70–77. 1934. no. 1.

A complete description of the various methods of solving equations of the second degree, and recommendations of the best pedagogic procedures. Applications of quadratic equations to geometry are made, and advice is given to teachers on the best methods of constructing real problems.

6. Risselman, W. C. *On the equiangular spiral.* School Science and Mathematics. 35: 55–62. January 1935.

A very clear and readable discussion of the theory, the properties and the application of this famous curve. The author points out the following interesting facts:

a. The equiangular spiral is involved in the theories of computation.
b. It represents the law of growth of bacteria and of human beings, according to Malthus.
c. It represents the compound interest law.
d. It occurs in nature.
e. It is connected, in music, with the equally tempered scale of the piano and, in art, with the theory of dynamic symmetry.

7. Snigireff, V. *Literal expressions.* Matematika i fisika v srednei shkole. (Mathematics and Physics in the secondary schools.) Quarterly Publication of the Commissariat of Education and Public Instruction of U.S.S.R. 1: 57–61. 1934 no. 1.

A discussion of the use of literal expressions in the arithmetic of the fifth grade and in the algebra of the sixth grade. The article contains, in addition, the outlines of model lessons on some of the topics discussed.

8. Struyk, Adrian. *Diophantine recreations.* School Science and Mathematics. 35: 269–72. March 1935.

The writer makes an analysis of the possibility of factoring $px^2-qx-t$ under the following conditions:

(a) $p$, $q$, and $t$ are consecutive integers.
(b) the factors must be linear with integral coefficients.

Put in more precise form, the problem is: To find integers $a, m, n, r, s$, such that $(a-1)x^2-ax-(a+1)=(mx+n)(rx-s)$.

To illustrate: By actual trial we discover that

$$4x^2-\ 5x-\ 6\equiv(4x+3)(x-2)$$
$$12x^2-13x-14\equiv(3x+2)(4x-7)$$

"What is desired, however, is a procedure more definite than experiment for determining the relationship between these numbers."

### Arithmetic

1. Bennett, E. *Integration of arithmetic in the upper elementary grades.* Virginia Journal of Education. 28: 255–57. March 1935.

The ideas kept constantly in mind have been how to make arithmetical facts and concepts meaningful to the children on the fifth, sixth, and seventh grade levels "so that they will function now in their social living, serve them adequately as they continue their study of mathematics and to care for some of the future mathematical needs in actual living situations."

After describing the many ways in which arithmetic is integrated and the various benefits accruing from the integration, the writer makes the following admission, which, to one reader at least, seems highly damaging to the exaggerated claims that the exponents have been making for their educational philosophy.

"Integration is practical and can be used often but it does not meet all of the needs to give the child the arithmetical knowledge necessary for him to have. . . . Thus as we do not yet

have satisfactory technique of integrated arithmetic to care for the child's needs, we must use 'common sense' and use drills, problem solving and any other need the subject demands, and use these separate and apart from the integrated program."

Such intellectual honesty should be commended; especially since it emanates from the cradle and hotbed of the movement for integration.

2. Connelley, R. L. *Diagnostic tests in arithmetic*. The Instructor. 44: 49+. January 1935.

The tests consist of 65 examples, examining the four fundamental operations with whole numbers, fractions and decimals, as well as the ability to solve problems.

The interesting feature of the test is a diagnostic chart which describes the specific skill each of the examples was intended to test. The teacher is advised to check on this chart "the number of each problem for which the pupil gave an incorrect response. Then by referring to the chart, she can easily tell the specific type of work on which explanation or drill is needed."

3. Crocker, G. H. *Number stories—fixing number facts through story-telling*. Grade Teacher. 52: 31+. March 1935.

Stories describing natural situations that will convince children of the desirability of learning to tell time and of knowing the number of days in a week.

4. Madison, H. *Rummage sale*. Wisconsin Journal of Education. 67: 267–8. February 1935.

A description of the way in which a rummage sale of the toys made in the art class was used to inculcate certain number facts and operations. "Aside from the increased skill in solving problems, which involved U. S. money, the children had improved in other ways as well, such as showing courtesy while shopping, making decisions quickly, and wrapping packages neatly."

5. Myers, G. C. *Number difficulties; common difficulties in arithmetic*. Grade Teacher. 52: 52–3. March 1935.

Specific advice on teaching the telling of time, the proper use of zeros and the concept of ratio.

6. Rich, Frank M. *Making arithmetic tables visual*. American Childhood. 20: 10–11. March 1935.

A description of numerous practical devices, such as home made spring balances, postal scales the clock, the thermometer, etc., that will make the arithmetic tables visual and thus pave the way for a real foundation in problem solving.

### Geometry

1. Cole, Lena R. *Note on introduction of area concept*. National Mathematics Magazine 9: 184. March 1935.

The writer recommends that the pupils will get a better concept of area "if the teacher have them cut out of cardboard quite a number of squares, one inch on a side, and actually use them as measuring instruments, just as their foot rulers. Have them see how many of these squares it takes to cover a book, or a notebook, or rectangles and squares drawn on paper."

2. Hill, Merton E. *The case of plane geometry*. California Journal of Secondary Education. 10: 107–10. January 1935.

Some good advice by the director of admission of the University of California, on teaching geometry as a meaningful subject.

3. James, W. C. *Observations on solid geometry*. Bulletin of the Kansas Association of Mathematics Teachers. vol. 9, no. 2, p. 7. February 1935.

A plea for closer cooperation between the teacher of solid geometry and the teacher of descriptive geometry.

4. McCoy, Dorothy. *Space*. National Mathematics Magazine. 9: 155–62. March 1935.

A discussion of the properties of n-dimensional space by means of the equations of analytic geometry. A good bibliography is appended.

5. Zagoskina, E. *Areas of rectilinear figures*. Matematika i fisika v srednei shkole. (Mathematics and Physics in the secondary schools.) Quarterly Publication of the Commissariat of Education and Public Instruction of U.S.S.R. 1: 66–69. 1934 no. 1.

The writer advocates the employment of models and the use of the concept of motion in the derivation of formulas for areas of rectilinear figures, and in the solution of practical problems. He believes that models should be made by the pupils at home as part of their assigned work, and at school in connection with the programs of the mathematical club.

6. Zetel, S. *Remarkable cases of non-congruence of triangles*. Matematika i fisika v srednei shkole. (Mathematics and Physics in the secondary schools.) Quarterly Publication of the Commissariat of Education and Public Instruction of U.S.S.R. 1: 10–14. 1934 no. 1.

The author points out that there are many pairs of non-congruent triangles that have two sides and three angles of one equal to two sides and three angles of the other. He also develops a simple technique for generating at will such sets of triangles. The following are examples of such pairs.

$$\left\{\begin{array}{l}27,\ 45,\ 75\\45,\ 75,\ 125\end{array}\right. \qquad \left\{\begin{array}{l}27,\ 36,\ 48\\36,\ 48,\ 64\end{array}\right.$$

$$\left\{\begin{array}{l}8,\ 12,\ 18\\12,\ 18,\ 27\end{array}\right. \qquad \left\{\begin{array}{l}64,\ 80,\ 100\\80,\ 100,\ 125\end{array}\right.$$

That the triangles in each pair are mutually equiangular can be seen at once by the fact that the sides are in proportion.

The wise teacher would do well by making use of these sets of triangles in his classes of geometry. For, not only will they be a source of wonder and excitement, but they will illustrate very pointedly the importance and meaning of the word "respectively," which all pupils use but few understand.

### Miscellaneous

1. Barkan, Samuel H. *The checking of homework in mathematics*. High Points. 17: no. 2. February 1935, pp. 44–45.

The writer describes an original method of checking homework in mathematics and points out the advantages thereof.

2. Barsukoff, A. *The course in mathematics in the secondary schools of France*. Matematika i fisika v srednei shkole. (Mathematics and Physics in the secondary schools.) Quarterly Publication of the Commissariat of Education and Public Instruction of U.S.S.R. 1: 86–91. 1934 no. 1.

A critical discussion of the subject matter and method of instruction of mathematics in the French secondary schools.

3. Chistakoff, I. *New researches in the field of ancient mathematics*. Matematika i fisika v srednei shkole.

(Mathematics and Physics in the secondary schools.) Quarterly Publication of the Commissariat of Education and Public Instruction of U.S.S.R. 1: 5–10. 1934 no. 1.

The writer relates the results obtained from the deciphering of the Moscow Papyrus, and discusses the work of Neugebauer on Babylonian mathematics.

4. Daugherty, R. D. *Concerning the construction and use of mathematical tables.* Bulletin of the Kansas Association of Mathematics Teachers. 9: no. 3, pp. 5–6. February 1935.

A brief historical sketch of the theory, construction and use of mathematical tables.

5. Dubislav, Walter. *Der universale Character der Mathematik.* (*The universal nature of mathematics.*) Unterrichtsblätter für Mathematik und Naturwissenschaften. (Journal for the Teaching of Mathematics and the Natural Sciences.) 41: 22–26. 1935 no. 1.

A discussion of the universal nature of mathematics with special reference to the work of Brouwer and Hilbert.

6. Emmons, C. H. *Mathematics, right or left?* National Mathematics Magazine. 9: 166–72. March 1935.

An exposition of the various experiments dealing with the modification of the requirements for entrance into the colleges.

7. Gwinner, Harry. *The two Wentworths.* National Mathematics Magazine. 9: 165. March 1935.

A short biographical sketch of the authors, father and son, of the famous series of text books in mathematics.

8. Irwin, J. O. *Some aspects of the development of modern statistical method.* Mathematical Gazette. 19: 18–30. February 1935.

A concise but very informative account of the development of modern statistical method.

9. Langer, Rudolph E. *Mathematics as an entrance requirement at the University of Wisconsin.* School Science and Mathematics. 35: 233–40. March 1935.

A description of the new status of mathematics and an analysis of the whole problem of its requirement as an entrance condition.

10. Lobko, I. A. *Notes on the history of the metric system.* Matematika i fisika v srednei shkole. (Mathematics and Physics in the secondary schools.) Quarterly Publication of the Commissariat of Education and Public Instruction of U.S.S.R. 1: 18–32. 1935 no. 1.

A comparative study of the spread of the metric system in Russia and in the other European countries.

11. *Der mathematische Unterricht im dritten Reich.* (*Mathematical instruction in the third Reich.*) Zeitschrift für mathematischen und naturwissenschaftlichen Unterricht aller Schulgattungen. (Magazine for the Teaching of Mathematics and the Natural Sciences in all school grades.) 66: 1–14. 1935 no. 1.

The report of a symposium in which eight speakers discussed various phases of mathematical education in contemporary Germany.

12. Miller, G. A. *History of mathematics in America.* School Science and Mathematics. 35: 292–96. March 1935.

A presentation of views which differ somewhat from those expressed by F. Cajori in *The Teaching and History of*

*Mathematics in the United States*, and by D. E. Smith and J. Ginsburg in their recent *History of Mathematics in America before 1900.*

"Teachers of mathematics who are interested in the development of their subject in our country may find it helpful to discuss with their students the merits of some discordant historical views relating thereto, since a deeper insight into the subject concerned can often be obtained in this way, and it is very desirable to cultivate the habit to accept historical statements relating to mathematics only after a careful consideration of their bearings."

13. Polya, G. *How to look for the solution.* National Mathematics Magazine. 9: 172–3. March 1935.

An interesting outline of a procedure to be followed in solving mathematical problems.

14. O'Neill, John J. *Machines invented to solve equations.* New York Herald Tribune. March 17, 1935. Section II, p. 8.

A description of three machines developed here and abroad for the solution of single equations and sets of equations. Through the courtesy of Mr. O'Neill, the science editor, the following references are given for those interested in more details.

Technology Review (M.I.T.)
Nature, January 12, 1935.
Nature, February 9, 1935.

15. Reeve, W. D. *Mathematics and the integrated program.* Teachers College Record. 36: 497–508. March 1935.

In discussing the general problem of integration, the writer makes the following important points:

a. We ought to come to some agreement as to what the word "integration" means.
b. "In attempting to correlate, subject matter teachers, and particularly administrators have made the mistake of trying to correlate by merely changing the names of courses, calling algebra and geometry 'correlated' mathematics, history and geography 'social studies,' and the like."
c. Some of our leading educators and scholars are not certain that the integration movement will furnish the best procedure for selecting that subject matter which harmonizes with the life experiences of the pupil.
d. " . . . before teachers can properly correlate mathematics with other fields, they ought to learn how to correlate the various parts of mathematics."
e. "It is conceivable (and perhaps desirable) that traditional organization of subject matter will be replaced by new learning units, which have been prepared to meet the needs of the pupils. But if this is done we will still have to guard against mechanical correlation. However, in most of the integrated programs, as now set up, subject matter is not utilized until needed. When this happens such large blocks of subject matter have to be brought in that the teacher, because of internal sequence of the content material, does the task mechanically for the pupils, and understanding and all sense of relationships between fields are lost. *Thus opportunistic teaching may have as great or even greater weaknesses than systematic development of subject matter.*" (Italics are the reviewer's.) A bibliography on mathematics and the integrated program, containing 50 references, is appended.

16. Trainor, Joseph, C. *A new approach for a course in mathematics.* School and Society. 14: 398. March 23, 1935.

A description of a course given to prospective teachers in which mathematics was treated as a mode of thinking. The topics taken up were those that are usually considered in a course on philosophy of mathematics or on fundamental concepts of mathematics.

The aims of the course were:

a. "The demonstration to the students that mathematics is a live subject to day.
b. "The pointing out that the most significant human thought which the race has is in its higher mathematics.
c. "To show that the processes of thought used in higher mathematics have significance for everyday thinking.
d. "To have the student become acquainted with some very important recent literature on the subject."

17. Wheeler, J. J. *Mathematics tomorrow—what and why.* Bulletin of the Kansas Association of Mathematics Teachers. 9: no. 3, pp. 1–5. February 1935.

A discussion of many topics, ranging from problems in mathematical philosophy to questions in curriculum making.

18. *Vorschläge für die Verdeutschung mathematischer Fachausdrücke.* (*Recommendations for the Germanization of technical, mathematical terms.*) Unterrichtsblätter für Mathematik und Naturwissenschaften (Journal for the Teaching of Mathematics and the Natural Sciences.) 40: 314–20. 1934 no. 9.

A list is given of about 250 technical terms that are mainly of Latin and Greek derivation. The german substitutes are given in an adjoining column and their use is highly recommended. The textbook writers are "requested" to make the proposed changes in terminology in their forthcoming textbooks in mathematics. The following is a partial list of some of the terms that have become interwoven with the very thought and speech of civilized mankind but which are to be thrown on the scrap heap and replaced by words of a purer origin:—Exponent, Geometrie, Infinitessimal, Koeffizient, Konstante, Maximum, Minimum, Polygon, Projektion, Radius, Asymptote, Symmetrie, Trigonometrie, Mantisse, Multiplikation, parallel, Logarithmus, Hypothenuse, Algebra, Arithmetic, plus, minus, positive, negative, and the word "mathematik."

That such a proposal should have come from the philologists is not at all surprising; but that it should have received the support of the leading mathematical educators in Germany is too painful to believe and too tragic to contemplate. Of all the sciences, mathematics, even more than physics, chemistry and biology is a truly international enterprise. It is therefore lamentable that a nationalistic movement should have been encouraged which can but lead to greater spiritual isolation and increased intellectual estrangement.

19. Zertcheninoff, N. *The conduct of examinations in mathematics.* Matematika i fisika v srednei shkole. (Mathematics and Physics in the secondary schools.) Quarterly Publication of the Commissariat of Education and Public Instruction of U.S.S.R. 1: 78–81. 1934 no. 1.

A discussion of the relative merits of oral and written examinations in the various branches of secondary mathe-

matics. It is interesting to note that the education law of Russia ***prescribes*** that every pupil must be given, at least once a semester, an oral test of no less than six minutes' duration, in addition to a two hour written examination.

20. Zimin, M. ***Trigonometric identities of the form*** $f(\sin x)=f(\cos x)$. Matematika i fisika v srednei shkole. (Mathematics and Physics in the secondary schools.) Quarterly Publication of the Commissariat of Education and Public Instruction of U.S.S.R. 1:14–15. 1934 no. 1.

A description of the method of deriving identities of the above type and of the form

$$f(\tan x)=f(\cot x).$$

# Annual Meeting of the National Council of Teachers of Mathematics

THE NEXT annual meeting of the National Council will be held at the Coronado Hotel, St. Louis, Mo., Dec. 31–Jan. 1, 1935–6. The explanation of the unusual date is that after the Council voted in February to hold a program meeting in St. Louis during the holidays in connection with the American Association for the Advancement of Science the N.E.A. selected St. Louis as the place of its February meeting for 1936. The Board of Directors, after due consideration, decided it would be unwise to hold two meetings in the same locality only seven weeks apart and voted to abandon plans for the usual February meeting and hold the regular annual meeting on the dates just mentioned. It is hoped that a greater number of high school teachers will be able to attend the meeting during the vacation period than would be possible in February.

The program is not completed. The sessions will begin at 9:30 A.M. Dec. 31, with a joint session with the Mathematical Association of America and Section A(Mathematics) of the A.A.A.S. At this session we shall hear and discuss a report of the joint commission of the M.A.A. and the Council which was appointed last February. This commission is to continue and extend the work done by the National Committee on the Reorganization of Secondary Mathematics, whose *Report* (1919) has done so much to improve the teaching of mathematics. The joint commission is also taking over most of the work of the Council's Policy Committee. The Council's representatives on the commission are William Betz, M. L. Hartung, G. H. Jamison, Ruth Lane, J. A. Nyberg, Mary A. Potter, and W. D. Reeve. Professor K. P. Williams of Indiana University is the chairman.

The joint session will also be addressed by Mr. William Betz, past president of the Council on some topic pertaining to the objectives and purposes of high school mathematics.

Negotiations are bing carried on for a joint session with Section Q(Education) of the A.A.A.S. in which it is proposed to discuss the problem of the dull pupil.

Other program sessions during the two days include the following

speakers and topics: Professor Wm. H. Roever, Washington University, St. Louis, on "The Purpose, Nature, and Use of Pictures in the Teaching of Geometry"; Mr. Rolland R. Smith, Springfield, Mass., on "Developing the Meaning of Demonstration in Geometry"; Professor Edwin W. Schreiber on "The History of the Metric System"[1] (Illustrated); and Miss Martha T. Denny, Oklahoma City, on "Modern Objective Tests."

The sessions will close as usual with the annual banquet, which is to be held in the Coronado Hotel, Wednesday evening, Jan. 1.

Mr. L. D. Haerrter, Principal of the John Burroughs School, St. Louis, is chairman of the St. Louis Local Committee.

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## Official Notice

**As secretary of the National Council of Teachers of Mathematics, I officially announce the annual election of certain officers of the National Council, said election to take place at St. Louis, Mo. on Wednesday, January 1, 1936. Article III Section 7, of the by-laws states: "At least two months before the date of the annual meeting, all members shall be given the opportunity through announcement in the official journal to suggest by mail for the guidance of the directors a candidate for each elective office for the ensuing year. At least one month before the annual meeting the secretary of the board of directors shall send to each member an official ballot giving the names of two candidates for each office to be filled. These candidates shall be selected by a nominating committee of the board of which the secretary shall be chairman. The election shall be by mail or in person and shall close on the date of the annual meeting."**

**At the Detroit meeting, 1931, of the National Council, the nominating committee consisting of the two most recent ex-presidents and the secretary as chairman (for this year: John P. Everett, William Betz, and Edwin W. Schreiber), was instructed to prepare a primary ballot suggesting five eligible candidates for each elective office. The officers to be elected at the St. Louis meeting are: President, 1936-38, second vice president, 1936-38 and three directors, 1936-39.**

**The periods of service of the officers of the National Council, from its organization in 1920 to the present time, are printed below.**

**EDWIN W. SCHREIBER,** ***Secretary***

# The National Council of Teachers of Mathematics

ORGANIZED 1920—INCORPORATED 1928

*Periods of Service of the Officers of the National Council*

PRESIDENTS

C. M. Austin, Oak Park, Ill., 1920
J. H. Minnick, Philadelphia, Pa., 1921–1923
Raleigh Schorling, Ann Arbor, Mich., 1924–1925
Marie Gugle, Columbus, Ohio, 1926–1927
Harry C. Barber, Exeter N.H., 1928–1929
John P. Everett, Kalamazoo, Mich., 1930–1931
William Betz, Rochester, N.Y., 1932–1933
J. O. Hassler, Norman, Okla. 1934–1935

VICE-PRESIDENTS

H. O. Rugg, New York City, 1920
E. H. Taylor, Charleston, Ill., 1921
Eula Weeks, St. Louis, Mo., 1922
Mabel Sykes, Chicago, Ill., 1923
Florence Bixby, Milwaukee, Wis., 1924
Winnie Daley, New Orleans, La., 1925
W. W. Hart, Madison, Wis., 1926
C. M. Austin, Oak Park, Ill., 1927–1928
Mary S. Sabin, Denver, Colo., 1928–1929
Hallie S. Poole, Buffalo, N.Y., 1929–1930
W. S. Schlauch, New York City, 1930–1931
Martha Hildebrandt, Maywood, Ill., 1931–1932
Mary A. Potter, Racine, Wis., 1932–1933
Ralph Beatley, Cambridge, Mass., 1933–1934
Allan R. Congdon, Lincoln, Nebr., 1934–1935
Florence Brooks Miller, Shaker Heights, Ohio, 1935–1936

SECRETARY-TREASURERS

J. A. Foberg, Chicago, Ill., 1920–1922, 1923–1926, 1927, 1928 (Appointed by Board of Directors)
Edwin W. Schreiber, Ann Arbor, Mich., and Macomb, Ill., 1929– (Appointed by the Board of Directors)

COMMITTEE ON OFFICIAL JOURNAL

John R. Clark, Editor, 1921–1928
W. D. Reeve, Editor, 1928–
Vera Sanford, 1929–
H. E. Slaught, 1928–

DIRECTORS

Marie Gugle, Columbus, Ohio, 1920–1922, 1928–1930, 1931–1933.
Jonathan Rorer, Philadelphia, Pa., 1920–1922
Harry Wheeler, Worcester, Mass., 1920–1921
W. A. Austin, Fresno, Calif., 1920–1921
W. D. Reeve, Minneapolis, Minn., 1920, 1926–1927
W. D. Beck, Iowa City, Iowa, 1920
Orpha Worden, Detroit, Mich., 1921–1923, 1924–1927
C. M. Austin, Oak Park, Ill., 1921–1923, 1924–1927, 1930–1932
Gertrude Allen, Oakland, Calif., 1922–1924
W. W. Rankin, Durham, N.C., 1922–1924

Eula Weeks, St. Louis, Mo., 1923–1925
W. C. Eells, Walla Walla, Wash., 1923–1925
*Harry English, Washington, D.C., 1925–1927, 1928–1930
Harry C. Barber, Boston, Mass., 1925–1927, 1930–1932, 1933–1935
Frank C. Touton, Los Angeles, Calif., 1926–1928
Vera Sanford, New York City, 1927–1928
William Betz, Rochester, N.Y., 1927–1929, 1930–1931, 1934–36
Walter F. Downey, Boston, Mass., 1928–1929
Edwin W. Schreiber, Ann Arbor, Mich., 1928–1929
Elizabeth Dice, Dallas, Tex., 1928, 1929–1931
J. O. Hassler, Norman, Okla., 1928, 1929–1931, 1933
John R. Clark, New York City, 1929–1931
Mary S. Sabin, Denver, Colo., 1929–1930, 1931–1933
J. A. Foberg, California, Pa., 1929
C. Louis Thiele, Detroit, Mich., 1931–1933
John P. Everett, Kalamazoo, Mich., 1932–1934
Elsie P. Johnson, Oak Park, Ill., 1932–1934
Raleigh Schorling, Ann Arbor, Mich., 1932–1934
W. S. Schlauch, New York City, 1933–1935
H. C. Christofferson, Oxford, Ohio, 1934–1936
Edith Woolsey, Minneapolis, Minn., 1934–1936
Martha Hildebrandt, Maywood, Ill., 1934–1935
M. L. Hartung, Madison, Wis., 1935–1937
Mary A. Potter, Racine, Wis., 1935–1937
Rolland R. Smith, Springfield, Mass., 1935–1937

* Deceased

# Official Primary Ballot

*The National Council of Teachers of Mathematics*
*St. Louis Meeting, January 1, 1936*

For President, 1936–38 (Vote for two)

( ) CONGDON, ALLAN R. Lincoln, Nebr.
( ) HILDEBRANDT, MARTHA Maywood, Ill.
( ) POTTER, MARY A. Racine, Wis.
( ) SANFORD, VERA Oneonta, N.Y.
( ) SCHLAUCH, W. S. New York, N.Y.

For Second Vice-President, 1936–38 (Vote for two)

( ) HERBERT, AGNES Baltimore, Md.
( ) JOHNSON, J. T. Chicago, Ill.
( ) KELLY, MARY Wichita, Kans.
( ) ORLEANS, JOSEPH B. New York, N.Y.
( ) MORTON, R. L. Athens, Ohio

For Members of the Board of Directors, 1936–39 (Vote for six)

( ) BRESLICH, E. R. Chicago, Ill.
( ) CHARLESWORTH, H. W. Denver, Colo.
( ) CHIPMAN, HOPE Ann Arbor, Mich.
( ) DENNY, MARTHA T. Oklahoma City, Okla.
( ) HAERTTER, LEONARD D. St. Louis, Mo.
( ) JAMISON, G. H. Kirksville, Mo.
( ) KEARNEY, DORA E. Ceder Falls, Iowa
( ) KEE, OLIVE A. Boston, Mass.
( ) LIDELL, B. W. Atlantic City, N.J.
( ) MALLORY, VIRGIL S. Montclair, N.J.
( ) OLDS, EDWIN G. Pittsburgh, Pa.
( ) PODMELE, THERESA Buffalo, N.Y.
( ) SHRINER, WALTER O. Terre Haute, Ind.
( ) WEIMAR, M. BIRD Wichita, Kans.
( ) WREN, FRANK L. Nashville, Tenn.

Members will please mark this ballot immediately and mail same to Edwin W. Schreiber, Secretary, 425 E. Calhoun St., Macomb, Illinois. Since our Annual Meeting this year will be six weeks earlier than usual the Final Ballot must appear in the November Issue of the *Mathematics Teacher*. That means vote NOW. Kindly place your name and address on the outside of the envelope. If you prefer to make a copy of this ballot on a separate sheet of paper it will be acceptable.